Lab: Curve Sketching

1. $f(x) = 4x^3 - 3x^4$

$f'(x) = 12x^2 - 12x^3$
$= 12x^2(1-x)$

f' ⟵ + 0 + 1 − ⟶

$f''(x) = 24x - 36x^2$
$= 12x(2-3x)$

f'' ⟵ − 0 + 2/3 − ⟶

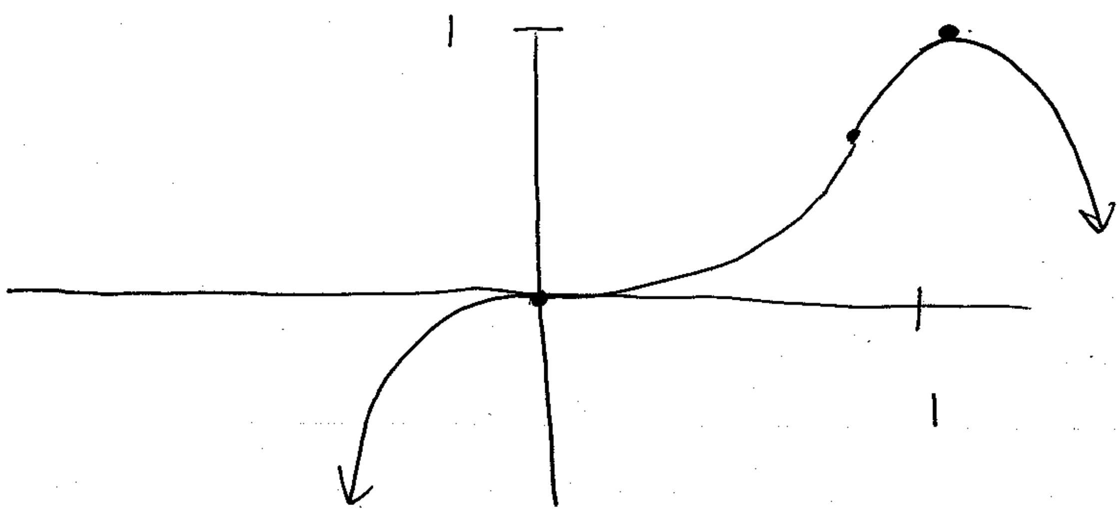
$\lim_{x \to \infty} 4x^3 - 3x^4 = -\infty$

$\lim_{x \to -\infty} 4x^3 - 3x^4 = -\infty$

$f(\frac{2}{3}) = 4 \cdot \frac{8}{27} - 3 \cdot \frac{16}{81}$
$= \frac{32}{27} - \frac{16}{27} = \frac{16}{27}$

D: $(-\infty, \infty)$

f inc on $(-\infty, 1)$ since $f' > 0$
f dec on $(1, \infty)$ since $f' < 0$
$1 = f(1)$ is rel. max; f' + to −
no rel min
f cc up $(0, 2/3)$; $f'' > 0$
f cc dn $(-\infty, 0) \cup (\frac{2}{3}, \infty)$; $f'' < 0$
inf. pts $(0,0)$ & $(\frac{2}{3}, \frac{16}{27})$
since f'' changes sign
No V.A.
No H.A.



2. $f(x) = 2x^3 + 3x^2 - 12x + 1$

$f'(x) = 6x^2 + 6x - 12$

$= 6(x^2 + x - 2)$

$= 6(x+2)(x-1)$

f' $\xleftrightarrow{\quad + \quad \underset{-2}{|} \quad - \quad \underset{1}{|} \quad + \quad}$

$f''(x) = 12x + 6$

f'' $\xleftrightarrow{\quad - \quad \underset{-1/2}{|} \quad + \quad}$

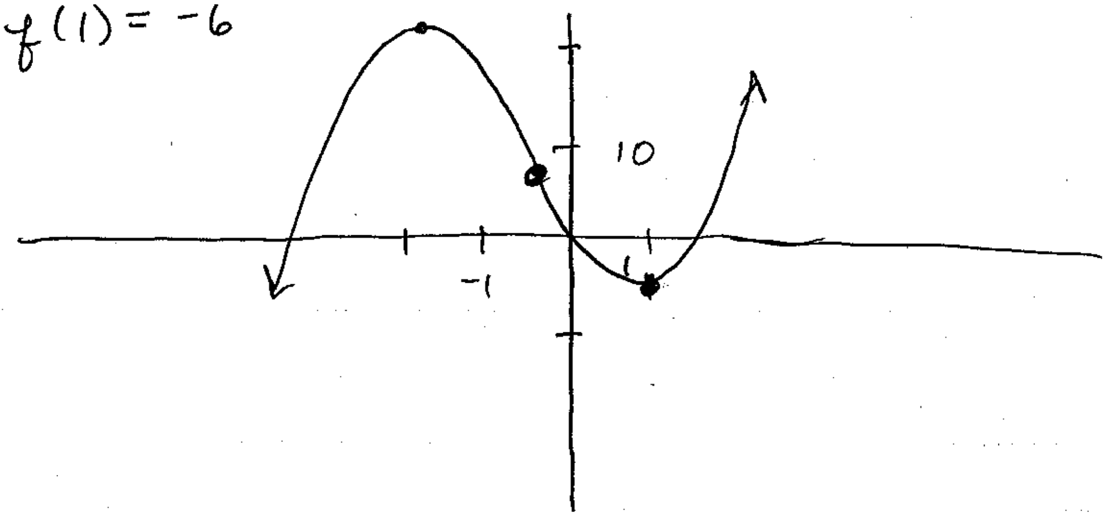
$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -\infty$

$$\begin{array}{r|rrr|r} -1/2 & 2 & 3 & -12 & 1 \\ & & -1 & -1 & \frac{13}{2} \\ \hline & 2 & 2 & -13 & \frac{15}{2} \end{array}$$

$$\begin{array}{r|rrr|r} -2 & 2 & 3 & -12 & 1 \\ & & -4 & 2 & 20 \\ \hline & 2 & -1 & -10 & 21 \end{array}$$

$f(1) = -6$



D: $(-\infty, \infty)$

f inc on $(-\infty, -2) \cup (1, \infty)$ since $f' > 0$

f dec on $(-2, 1)$ since $f' < 0$

$21 = f(-2)$ rel. max since f' changes + to −

$-6 = f(1)$ rel. min since f' changes − to +

f cc up on $(-\frac{1}{2}, \infty)$ since $f'' > 0$

f cc down $(-\infty, -\frac{1}{2})$ since $f'' < 0$

inf point $(-\frac{1}{2}, \frac{15}{2})$ since f'' changes sign

no vertical asymp.

no horizontal

3. $G(x) = \dfrac{x}{x^2+4}$

$G'(x) = \dfrac{1\cdot(x^2+4) - 2x\cdot x}{(x^2+4)^2}$

$= \dfrac{-x^2+4}{(x^2+4)^2}$

G' $\longleftarrow$ $-$ | $+$ | $-$ $\longrightarrow$

-2 $\quad$ 2

$G''(x) = \dfrac{-2x(x^2+4)^2 - 2(x^2+4)(2x)(-x^2+4)}{(x^2+4)^4}$

$= \dfrac{(x^2+4)[-2x(x^2+4) - 4x(-x^2+4)]}{(x^2+4)^3}$

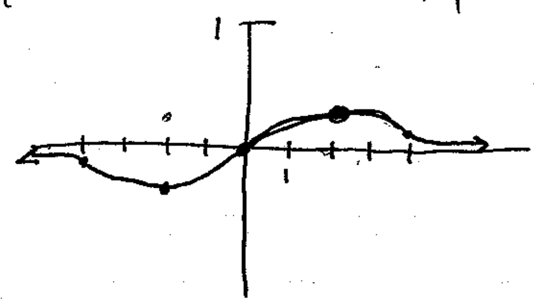
$= \dfrac{-2x^3 - 8x + 4x^3 - 16x}{(x^2+4)^3}$

$= \dfrac{2x^3 - 24x}{(x^2+4)^3} = \dfrac{2x(x^2-12)}{(x^2+4)^3}$

G'' $\longleftarrow$ $-$ | $+$ | $-$ | $+$ $\longrightarrow$

$-2\sqrt{3}$ $\quad$ 0 $\quad$ $2\sqrt{3}$

$\lim\limits_{x \to -\infty} \dfrac{x}{x^2+4} = 0$; $\lim\limits_{x \to \infty} \dfrac{+x}{x^2+4} = 0$



D: $(-\infty, \infty)$

G inc $(-2,2)$ since $G' > 0$

G dec $(-\infty,-2)\cup(2,\infty)$ since $G' < 0$

$\frac{1}{4} = G(2)$ rel max G' + to −

$-\frac{1}{4} = G(-2)$ rel min G' − to +

G cc up $(-2\sqrt{3}, 0)\cup(2\sqrt{3},\infty)$ since $G'' > 0$

G cc dn $(-\infty,-2\sqrt{3})\cup(0,2\sqrt{3})$ $G'' < 0$

$\left(-2\sqrt{3}, -\frac{2\sqrt{3}}{16}\right)$
$\left(2\sqrt{3}, \frac{2\sqrt{3}}{16}\right)$ inf pts
$(0, 0)$ G'' sign change

No vert. asymp.

H.A.: $y = 0$

4. $H(x) = 4x^{1/2} + 4x^{-1/2}$

$H'(x) = 2x^{-1/2} - 2x^{-3/2}$

$= 2x^{-3/2}(x-1)$

H': 0 − 1 +

$H''(x) = -x^{-3/2} + 3x^{-5/2}$

$= x^{-5/2}(-x+3)$

H'': 0 + 3 −

$\lim_{x \to 0^+} \left(4\sqrt{x} + \frac{4}{\sqrt{x}}\right) = \infty$

$\lim_{x \to \infty} \left(4\sqrt{x} + \frac{4}{\sqrt{x}}\right) = \infty$

($\lim_{x \to -\infty} H(x)$ not possible)

$H(1) = 4 + 4 = 8$

$H(3) = 4\sqrt{3} + 4/\sqrt{3} = \frac{4}{\sqrt{3}}(3+1) = \frac{16}{\sqrt{3}}$

D: $(0, \infty)$

H inc $(1, \infty)$ since $H' > 0$

H dec $(0, 1)$ since $H' < 0$

no rel max

$8 = H(1)$ rel. min H' − to +

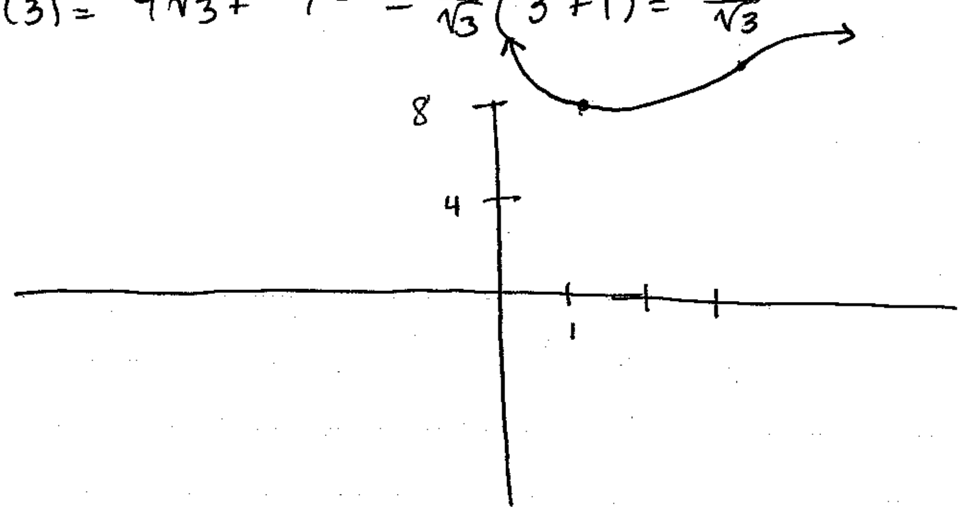
H cc up $(0, 3)$; $H'' > 0$

H cc dn $(3, \infty)$; $H'' < 0$

$(3, \frac{16}{\sqrt{3}})$ is inf. pt since H'' changes sign

$x = 0$ is V.A.

no H.A.



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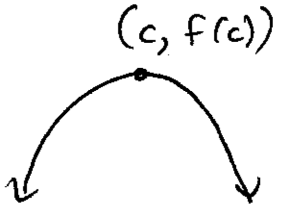

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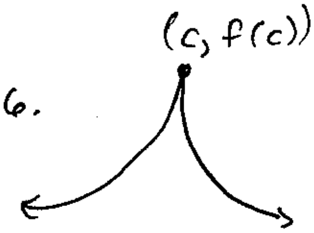

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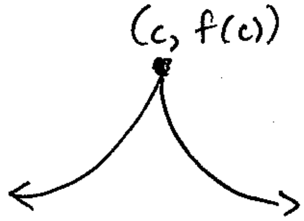

8. $f'(x) = 3ax^2 + 2bx$

$f''(x) = 6ax + 2b$

$f''(1) = 6a + 2b = 0$

$6a = -2b$

$-3a = b \qquad \cancel{b = 3/2}$

$f(1) = 2 \Rightarrow a + b = 2$

$-2a = 2 \qquad \cancel{4a = 2} \Rightarrow \cancel{a = 1/2}$

$a = -1$

$b = 3$

9. $f(0) = 0 \Rightarrow e = 0$

f is even $\Rightarrow b = d = 0$

$f(x) = ax^4 + cx^2$

$f'(x) = 4ax^3 + 2cx$

$f''(x) = 12ax^2 + 2c$

$f''(1) = 12a + 2c = 0$

$6a = -c$

$f(1) = a + c = -1$

$a - 6a = -1$

$-5a = -1$

$a = +1/5, \ c = -6/5$

10. $f'(x) = 2ax + b$

$f'(1) = 0 = 2a + b$

$-2a = b$

$f(1) = a + b + c = 7$

$-[f(2) = 4a + 2b + c = -2]$

$-3a - b = 9$

$-a = 9$

$a = -9$

$b = 18$

$-9 + 18 + c = 7$

$c = -2$

11. a. $f' > 0$ on $(-4,-2) \cup (0,3) \Rightarrow f$ inc $(-4,-2) \cup (0,3)$

b. $f' < 0$ on $(-\infty,-4) \cup (-2,0) \cup (3,\infty) \Rightarrow f$ dec $(-\infty,-4) \cup (-2,0) \cup (3,\infty)$

c. f has rel. max at $x = -2, 3$ since f' changes + to −

d. f has rel. min at $x = -4, 0$ since f' changes − to +

e. f cc up when $f'' > 0$ so f' inc $\Rightarrow (-\infty,-3) \cup (-1, 1.5)$

f. f cc dn when $f'' < 0$ so f' dec $\Rightarrow (-3,-1) \cup (1.5, \infty)$

g. f has inf points when f'' changes sign, so f' changes inc to dec or dec to inc $\Rightarrow x = -3, -1, 1.5$