AP Calculus
Curve Sketching and Graphing Calculators

Use your graphing calculator to complete each problem. Explain all answers.

For the function $y = e^x - x^8$ on [-2, 2] complete the following.

a. Find the x-intercepts.

$e^x - x^8 = 0$

$x = -.894,\ 1.155$

b. Find the x-coordinates of all relative extrema. Classify each as max or min.

y' changes sign $\quad$ y' changes + to −

$y' = 0$ if $x = .8374\ldots$ $\quad \Rightarrow x = .837$ produces rel. max

c. Find the relative extrema. Are there any absolute extrema?

rel. max is 2.068

$y(2) = -248.6\ldots$; $y(-2) = -255.8\ldots$ $\quad \Rightarrow$ abs max is 2.068, abs min is −255.864

d. Find the x-coordinates of all inflection points.

y'' changes sign at $x = -.472\ldots$ and $x = .561\ldots$

e. Find the point(s) of inflection.

$(-.472, .620)$
$(.561, 1.743)$

For the function whose derivative is given by $f'(x) = x^5 + \ln x$, answer the following questions for f(x). Explain all answers.

a. Find the x-coordinates of any relative extrema of f. Classify as max or min.

f' changes − to + at $x = .766 \Rightarrow f$ has rel min at .766

b. Does f have an absolute max or min? Why or why not?

f has only one critical number and since f dec then inc, f has abs min

c. Find the x-coordinates of any inflection points of f. Explain.

$f'(x)$ always increases $\Rightarrow f''(x)$ is always positive, so f is always concave up ; no inflection points

AP Calculus
WS: Graphing Calculators and Curve Sketching

Use your graphing calculator to complete each problem. Support your answers. Final answers must be accurate to 3 decimal places.

1. If $f(x) = x^3 - 5x^2 + x^5 + 2x$, find the following characteristics of f.

a. Find the x-coordinates of all relative extrema. Classify as max or min. Explain.

$f'(x) = 3x^2 - 10x + 5x^4 + 2$ always exists

$f'(x) = 0$ if $x = .214\ldots, 1$

$x = .214$ produces rel max f' + to −

$x = 1$ produces rel. min f' − to +

b. Find all relative extreme values.

rel max is $f(.214\ldots) = .209$

rel min is $f(1) = -1$

c. Find the x-coordinate of all inflection points. Explain.

$f''(x) = 6x - 10 + 20x^3$

$f''(x) = 0$ if $x = .668$ and f'' changes sign $\Rightarrow x = .668$

d. Find all inflection points.

$(.668, -.466)$

2. Given $f'(x) = x^3 - 5x^2 + \sin(x^2)$, find the following information for f(x). Explain all answers.

a. On what interval(s) does f increase?

$\cancel{f'(x) = 3x^2 - 10x + \cos(x^2)\cdot 2x > 0} \Rightarrow$

$f'(x) > 0$

$(5.003, \infty)$

b. On what interval(s) does f decrease?

$f'(x) < 0$

$(-\infty, 0) \cup (0, 5.003)$

c. On what interval(s) is f concave up?

$f'(x)$ inc or $f''(x) > 0$

$f''(x) = 3x^2 - 10x + \cos(x^2)\cdot 2x$

$(-\infty, 0) \cup (3.987, \infty)$

d. On what interval(s) is f concave down?

$f''(x) < 0$

$(0, 3.987)$

e. What are the x-coordinate(s) of the point(s) of inflection?

$x = 0, 3.987$ since $f''(x)$ changes sign

3. f is a continuous function on [-2, 4] so that f(-2) = 5 and f(4) = 1. The table below gives characteristics of the **derivative** of f. Answer each question. Justify.

x	-2 < x < 0	x = 0	0 < x < 2	x = 2	2 < x < 4
f '	positive increasing	DNE	negative increasing	0	negative decreasing

a. Find the x-coordinates of the relative extrema of f.

f has rel. min if f' changes - to + ⇒ no rel. min

f has rel. max if f' changes + to - ⇒ x=0 produces rel. max

b. Find the x-coordinates of the inflection point(s) of f.

f has an inflection point when f'' changes sign, or when the slope of f' changes sign, so when f' changes inc to dec or dec to inc ⇒ x=2

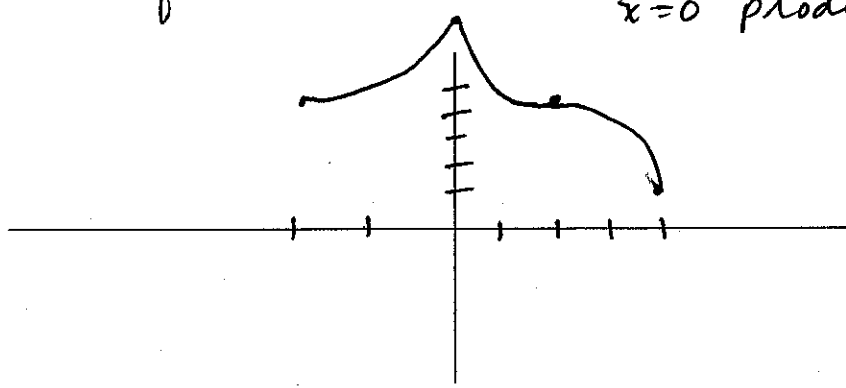
c. Find the x-coordinates of the absolute extrema of f.

f(-2)=5 and f(0) > f(-2) since f inc on (-2,0) because f' > 0 on (-2,0) ⇒ x=4 produces abs. min

f(4)=1

x=0 produces abs. max

d. Sketch the graph of f.



f' inc → f''>0 → f ccup on (-2,0) and (0,2)

f' dec → f''<0 → f cc dn on (2,4)