AP Calculus
Comprehension Checks: Chapter 4 Part 1

Extreme Values

1. Extreme values (maximum and minimum function values) are (x-values, y-values, or ordered pairs).
2. Define each term:
   a. absolute or global extrema $f(x) \geq f(c)$ for all x or $f(x) \leq f(c)$ for all x
   b. relative or local extrema In a two sided interval about c, f(c) is either a max or min
3. If a function is continuous on [a, b], then there MUST be what types of extrema of f on the interval? absolute
4. Endpoints of a closed interval are candidates for which type of extrema only? absolute
5. What type of extrema require a two-sided interval? local
6. To find extrema of a continuous function f on [a, b]:
   a. Determine the critical numbers of f by considering what TWO cases? $f' = 0$ or DNE
   b. Evaluate f at what x-values? endpoints & critical numbers
   c. How do you identify the absolute maximum and minimum? largest and smallest y-values from step 2

Rolle's Theorem and the Mean Value Theorem

1. Both Rolle's Theorem and the Mean Value Theorem require a function f that is continuous on [a, b] and differentiable on (a, b). Rolle's Theorem has the additional hypothesis that $f(a) = f(b)$.
2. If the hypotheses stated in question 1 are satisfied, the Mean Value Theorem GUARANTEES a value c located $a < c < b$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. Rolle's Theorem GUARANTEES a value c, $a < c < b$ such that $f'(c) = 0$.
3. Geometrically, both theorems find values of c where the tangent to the curve at x = c is parallel to the secant connecting $(a, f(a))$ and $(b, f(b))$
4. Both theorems locate x-values where the instantaneous rate of change at a particular point is equal to the average rate of change over the entire interval.

Curve Sketching

1. Find the domain of the function, which is all values of x for which the function produces a real result.
2. Find all critical values of the function, which are found by finding where the derivative of the function is 0 and DNE.
3. If a function is increasing on an interval, then its derivative is positive on the interval; if a function is decreasing on an interval, then its derivative is negative on the

interval; if a function is constant on an interval, then its derivative is ____0____ on that interval.

4. Use a ____labeled____ number line for ____f'____ to justify the intervals of increase and decrease for f. This is NOT known as the first derivative test.

5. The First Derivative Test states that if f is continuous on an open interval I that contains c, a critical number of f, and if f is differentiable on I, except possibly at c, then f(c) is a
   a. relative minimum value of f if f ' changes ____− to +____ at x = c
   b. relative maximum value of f if f ' changes ____+ to −____ at x = c

6. A function is concave up on an interval if the tangent lines to the curve are __below__ the curve and a function is concave down on an interval if the tangents to the curve are __above__ the curve.

7. If a function f is concave up on I, then f " is __+__ and f ' is __inc__ on I. If a function f is concave down on I, then f " is __−__ and f ' is __dec__ on I.

8. Use a ____labeled____ number line for ____f''____ to justify the intervals of concavity for f. This is NOT known as the second derivative test.

9. An inflection point is an ordered pair where ____Concavity____ of the function changes, so the second derivative of the function ____changes sign____.

10. The Second Derivative Tests states that f is a function such that f '(c) = 0 and f " exists on an open interval containing c. If
    a. f "(c) > 0, then f(c) is a ____rel. min____,
    b. f "(c) < 0, then f(c) is a ____rel. max____, and
    c. f "(c) = 0, then no conclusion about relative extrema is possible so the ____f' number line____ should be used to determine if a maximum or minimum has been located.

11. In order to show that a relative extrema has been found, it is NOT enough to show a zero or nonexistent first derivative and in order to show that a concavity change has been located, it is NOT enough to show a zero or nonexistent second derivative. Both require showing a ____sign change____ in the appropriate number line. The number line to use for showing a relative extreme value is __f'__ and for showing a concavity change the number line is __f''__. In either case, ____label____ all number lines.

12. The graph of f ' is provided to you. The graph of f will increase when the given graph ____is above the x-axis____ and the graph of f will decrease when the given graph ____is below the x-axis____. The graph of f will have a horizontal tangent when the given graph ____has an x-intercept____. The graph of f will have an inflection point when the given graph ____turns____. The graph of f will be concave up when the given graph

Increases and the graph of f will be concave down when the given graph decreases. The graph of f " will be above the x-axis when the given graph increases and the graph of f " will be below the x-axis when the given graph decreases.

13. When using an f, f ', or f " graph to answer questions about a related graph, all justifications MUST refer to the given graph.

14. To find horizontal asymptotes, evaluate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. If either or both of these limits are finite values L, then $y=L$ is a horizontal asymptote of f.

15. To find vertical asymptotes, find values of x so that either the right or left hand limit is $\pm\infty$. The vertical asymptotes are $x=k$ where k produces an infinite limit, either from the right, the left, or both.

Related Rates

1. Use mathematical notation to state the given and the quantity to be found. Be careful with signs. What words will indicate a positive rate of change? growing, inflating, increasing, etc. What words will indicate a negative rate of change? Shrinking, deflating, decreasing, etc.
2. Draw and label a diagram and state the equations relating all quantities. Remember to look for similar Δs, right Δs
3. Differentiate your equations with respect to the same variable, which is usually time and then substitute values and given information.
4. Don't forget to use units if provided in the question.
5. Write a sentence to give meaning to your answer. Change signs to a verbal context.

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Optimization

1. Define any variables introduced. State equations relating all quantities involved. State the quantities to be determined. Determine domain.
2. If there are more than two variable quantities, eliminate until only 2 remain. If radicals are involved, consider the square of the quantity. Maximums and minimums will be located at the same x-values if all quantities involved are positive.
3. Differentiate and solve for Critical numbers. Remember to check for nonexistent derivatives.
4. Use either the First Deriv. Test or Second Deriv. to classify the type, if any, of relative extrema located. Keep domain in mind.
5. Label all work!
6. Answer the question that was asked.

Differentials/Linearizations/Tangent Line Approximations

1. The "nice" value of x is one for which the function and its derivatives are easy to compute. This is known as c and should be "close" to the x-value at which you are approximating the function value.
2. The formula for the linearization or the tangent line approximation for f(x) at x = c is given by $L(x) = f(c) + f'(c)\cdot(x-c)$
3. The formula for the differential of y is given by $dy = f'(x)\cdot dx$.
4. The formula for the differential of y when x = c is given by $dy = f'(c)\cdot dx$.
5. Using differentials, "bad" y is approximately $y + dy$.
6. When computing estimates with differentials, dx = $x - c$ (bad x - nice x)
7. To determine if a differential/tangent line approximation is too large or too small, use the concavity of the original function when x = c. If f ″(c) > 0 then the tangent line estimates are too small because the tangent lines will be below the curve. If f ″(c) < 0 then the tangent line estimates are too large because the tangent lines will be above the curve.
8. The true value of Δy, also known as error or propagated error, is estimated by dy.
9. Relative error is approximated by $\frac{dy}{y}$ (nice y).
10. Percent error is approximated by rel. error × 100%

Business/Economics Applications

1. The demand function gives the price the company can charge.
2. The revenue function is $x \cdot p(x)$.
3. The profit function is $R - C$.
4. The average cost function is $C(x)/x$.
5. The break even point is where R and C are equal, so profit is 0.
6. Marginal means derivative. Marginal revenue/cost/profit for c units produced and sold is approximately the extra revenue/cost/profit from selling/producing/selling and producing the next item.

Newton's Method

1. State the formula for the n plus first approximation of the root of f(x).
   $x_{n+1} = x_n - f(x_n)/f'(x_n)$
2. If x_1 is not provided, use the graph to find an interval that contains a zero of the function.
3. The n plus first approximation is the x-intercept of the tangent line drawn to f(x) at the nth approximation.
4. Newton's method will fail if the tangent line to f(x) at x_1 is horizontal because finding the value of x_2 will involve division by 0.
5. Continue applying Newton's method until the desired accuracy is reached. This will be when two successive approximations agree to desired accuracy.
6. Record each approximation given by the calculator. If the question is graphical, sketch each tangent line and identify x_1, x_2, and so forth on the x-axis.