AP Calculus
Comprehension Checks: Chapter 10

Differential Equations

1. How do you determine the order of a differential equation? highest order derivative
2. What differentiates a general solution to a differential equation from a particular solution to a differential equation? general: +C ; particular: C is determined
3. How do you verify that a function is a solution to a differential equation? Take indicated deriv. & substitute
4. What type of differential equation can we solve in this course? Seperable, first order
5. What is the technique for solving the type of differential equation described in question 4? separation of varaables
6. When must you write your solution in function notation? When directions indicate $y=f(x)$ form

Slope Fields

1. What does a slope field allow you to see? picture of solution curves
2. How do you graph a slope field? draw small tangent line segments using $\frac{dy}{dx}$ expression
3. How do you match a slope field to a selection of differential equations? use info like $m=0$, m undefined

Euler's Method

1. What is the purpose of Euler's Method? determine estimates of points on sol. curve
2. What must you be given to use the technique? diff eq. and initial cond.
3. How do you get a more accurate estimate using Euler's Method? use a larger n
4. What are the table headings used in Euler's Method? x, y, m, dy
5. What is the first column you complete? x
6. How do you generate successive y-values? add y to dy
7. YOU WILL HAVE TO KNOW HOW TO SET UP AND HEAD THE COLUMNS OF YOUR TABLES.

Exponential Growth and Decay

1. State both forms of the mathematical model for exponential growth and decay. $\frac{dy}{dt} = ky$; $y = Ce^{kt}$
2. What case(s) result in growth? $k > 0$
3. What case(s) result in decay? $k < 0$
4. For the quantity being measured, what phrases indicate exponential growth or decay? half life, double, triple
5. For the rate of change of the quantity being measured, what phrase indicates exponential growth or decay? rate of change is proportional to dependent variable
6. Be careful of rounding errors!

Logistic Growth and Decay

1. What differential equation describes logistic growth or decay? $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$
2. What is the carrying capacity? L
3. What is the equation horizontal asymptote of the graph of t versus P? $P = L$

4. What is $\lim_{t\to\infty} P(t)$? L

5. When will the graph of t versus P show decay? $C > L$ (C is initial value $P(0)$)

6. When will the graph of t versus P show growth with no inflection point? $\frac{L}{2} < C < L$

7. When will the graph of t versus P show growth with an inflection point? Where is the inflection point located? $0 < C < \frac{L}{2}$; $P = \frac{L}{2}$

8. If you must solve a logistic differential equation, after you separate the variables, what technique will be used? separation of variables
partial fractions and $|A| = |B|$
ln will be antiderivatives and the ln terms will be subtracted

BC Calculus
Comprehension Checks Chapter 11

Parametrically Defined Curves

1. If x and y are functions of t, what expression gives dy/dx? $\frac{dy}{dt} \Big/ \frac{dx}{dt}$
2. What does dy/dx find for the curve described by x = f(t) and y = g(t) for a particular value of t? Slope of path
3. What variable will be used in the answer to 23? t
4. What expression gives d^2y/dx^2? $\frac{d}{dt}\left[\frac{dy}{dx}\right] \Big/ \frac{dx}{dt}$
5. How will you locate horizontal tangents for a curve defined parametrically in terms of t? $\frac{dx}{dt} \neq 0$ and $\frac{dy}{dt} = 0$
6. How will you locate vertical tangents for a curve defined parametrically in terms of t? $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$

Areas of Parametrically Defined Curves

1. Draw the representative rectangle and label endpoints in terms of x and y. Decide if one dimension is dx or dy. Indicate direction of movement.
2. If the curve is x = f(t), y = g(t), what expression gives dx? $f'(t)\,dt$
3. What expression gives dy? $g'(t)\,dt$
4. How do you determine limits of integration? direction of motion to cause left → right or bottom → top

Volumes of Solids of Revolution: Parametric

1. Draw the representative rectangle and label endpoints in terms of x and y. Decide if one dimension is dx or dy. Indicate direction of movement.
2. If the curve is x = f(t), y = g(t), what expression gives dx? $f'(t)\,dt$
3. What expression gives dy? $g'(t)\,dt$
4. How do you determine limits of integration? direction of motion for left → right or bottom → top
5. Disk/washer or shell may be used. Label r, h, outer, inner, and/or w in terms of x and y. Convert to the parameter using 2 and 3.

Arc Length for Parametrics

1. State the integrand for arc length of the length defined by x = f(t), y = g(t) from t = a to t = b. $\int_a^b \sqrt{(f'(t))^2 + (g'(t))^2}\,dt$
2. Direction of movement is not used in determining limits of integration. Use $a < b$

Vector Valued Functions

1. What are the 3 notations used to denote a vector valued function?
   - $\langle \ , \ \rangle$
   - $\vec{R}(t) =$
   - $f(t)\,\hat{i} + g(t)\,\hat{j}$
2. What additional notation can be used in print? Bold
3. What is the velocity vector? $\langle f'(t), g'(t) \rangle$

4. What is the speed of the particle at time t? $\sqrt{(f'(t))^2+(g'(t))^2}$

5. What is the acceleration vector? $\langle f''(t), g''(t)\rangle$

6. What is the distance traveled by the particle from t = a to t = b? $\int_a^b \sqrt{(f'(t))^2+(g'(t))^2}\,dt$

7. How do you graph the path of a particle given a position vector? use parametric techniques

8. What does dy/dx represent? Slope of path of object in xy-plane

9. Does dy/dx have anything to do with speed? No!

Conversion Equations between Polar and Cartesian

1. $x = r\cos\theta$; $y = r\sin\theta$
2. $x^2 + y^2 = r^2$; $\tan\theta = y/x$

Types of Polar Curves

1. Lines: $\theta = \theta_1$, $r = a\sec\theta$, $r = a\csc\theta$
2. Roses: $r = a\cos b\theta$ or $r = a\sin b\theta$
   a. Graph always contains pole.
   b. Petal length is $|a|$.
   c. Number of petals is b if b is odd, 2b if b is even.
   d. Petals are symmetric about the pole.
3. Cardioids/Limaçons: $r = a + b\sin\theta$, $r = a + b\cos\theta$

   Find all possible values of r.
   a. Cardioid with no loop if $[0,+]$ or $[-,0]$.
   b. Cardioid with loop if $[-,+]$.
   c. Convex cardioid that surrounds the pole but does not contain the pole if $[+,+]$ or $[-,-]$
4. Lemniscates: $r^2 = a\cos b\theta$ or $r^2 = a\sin b\theta$.
   a. Petal length is $\sqrt{|a|}$.
   b. Graph always contains pole.
   c. Number of petals is b if b is even, 2b if b is odd.

Area of Polar Curves

1. Sketch the curve. Use symmetry whenever possible.
2. Indicate region described in problem.
3. Watch direction of motion.
4. Key formula is $\int_\alpha^\beta \frac{1}{2}r^2\,d\theta$.

Slope of Polar Curves

1. Use conversion equations from the first section and the product rule to find dx/dθ and dy/dθ.
2. Slope is $\frac{dy}{d\theta} \Big/ \frac{dx}{d\theta}$.