AP Calculus
Comprehension Checks: Chapter 7

Derivatives of Inverse Functions

1. What is the algebraic definition for f and g being inverse functions? $f(g(x)) = g(f(x)) = x$
2. How are the domains and ranges of f and g related if f and g are inverse functions? they swap
3. If $g = f^{-1}$, how are the derivatives of f and g related? $g'(x) = \frac{1}{f'(g(x))}$
4. If $g = f^{-1}$, an x-value for g is what type of value for f? y-value

Natural Log

5. What is the derivative with respect to x of the function ln |x|? $\frac{1}{x}$
6. What is the process of logarithmic differentiation and when can you use it? log both sides, then differentiate; var Var or nasty fraction
7. What is the indefinite integral of 1/x with respect to x? $\ln|x| + C$
8. What is the indefinite integral of tangent x with respect to x? $\ln|\sec x| + C$ or $-\ln|\cos x| + C$
9. What is the indefinite integral of secant x with respect to x? $\ln|\sec x + \tan x| + C$ ← cos x
10. What is the indefinite integral of cotangent x with respect to x? $\ln|\sin x| + C$ or $-\ln|\csc x| + C$
11. What is the indefinite integral of cosecant x with respect to x? $-\ln|\csc x + \cot x| + C$
12. What is the process of logarithmic differentiation? ~~Log~~ ln both sides, then diff.
13. What must you remember in using logarithmic differentiation? mult. by y in its explicit form
14. When should you use logarithmic differentiation? Variable Variable or fraction

Exponential Functions

15. What is the derivative with respect to x of $e^x$? $e^x$
16. What is the indefinite integral of e to the x with respect to x? $e^x + C$
17. What is the indefinite integral of e to the ax power with respect to x? $\frac{1}{a} \cdot e^{ax} + C$
18. What are the conditions on a in the previous formula? a is constant

Bases other than e

19. What is the derivative of a to the x with respect to x? $a^x \cdot \ln a$
20. What is the indefinite integral of a to the x with respect to x? $\frac{a^x}{\ln a} + C$
21. What is the derivative of $\log_a x$ with respect to x? $\frac{1}{x \ln a}$
22. What are the restrictions on the value of a in the previous three formulas? a is constant, $a > 0$, $a \neq 1$
23. What precalculus rules for manipulating exponential and logarithmic expressions must you remember?

a. $\ln(ab) = \ln a + \ln b$
b. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
c. $\ln(a^b) = b \ln a$
d. $a^{\log_a(\_\_\_)} = \_\_\_\_$
e. $\log_a a^{\_} = \_$

f. $\log_a b = \frac{\log_c b}{\log_c a}$

g.

h.

i.

j.

Inverse Trig

24. What is the range of the inverse sine function? $[-\pi/2, \pi/2]$
25. What is the range of the inverse tangent function? $(-\pi/2, \pi/2)$
26. What is the range of the inverse secant function? $[0, \pi/2) \cup [\pi, 3\pi/2)$
27. What is the derivative with respect to x of the inverse sine function? $\frac{1}{\sqrt{1-x^2}}$
28. What is the derivative with respect to x of the inverse tangent function? $\frac{1}{1+x^2}$
29. What is the derivative with respect to x of the inverse secant function? $\frac{1}{x\sqrt{x^2-1}}$
30. What is the antiderivative of one over the square root of the difference of a squared and x squared with respect to x? $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right)+C$
31. What is the antiderivative of one over the sum of a squared and x squared with respect to x? $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$
32. What is the antiderivative of one over the product of x and the square root of the difference of x squared and a squared? $\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C$

L'Hopital's Rule

33. L'hopital's rule is a limit evaluation technique that may be used when both numerator and denominator are what by direct substitution? $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$
34. L'hopital's rule may be applied multiple times as long as what happens? $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$ by direct sub
35. Other indeterminate forms requiring l'hopital are
    a. $0\cdot\pm\infty$ in which case use ÷ by a reciprocal
    b. $\infty - \infty$ in which case use factor or common denom.
    c. $1^\infty$, $0^0$, or $0^\infty$ in which case change to base e

    All of these cases ultimately result in finding one of the original l'hopital cases.
36. L'hopital's rule fits in curve sketching as a technique to find what characteristics? asymptotes, types of discontinuity

Trigonometric Integrals

37. If the integrand has an odd power of sin or cos, what identity will you have to use in order to evaluate the integral? $\sin^2 x + \cos^2 x = 1$
38. If the integrand has an odd power of sin or cos, how will you use the identity to perform a u-substition? save one for du, switch all to other funct.
39. State the version of the double angle formula for cosine that is used to integrate $\sin^2 x$.
40. State the version of the double angle formula for cosine that is used to integrate $\cos^2 x$.
41. What identity relates $\tan^2 x$ to $\sec^2 x$?

39. $\sin^2 x = \frac{1-\cos(2x)}{2}$ 40. $\cos^2 x = \frac{1+\cos(2x)}{2}$ 41. $\tan^2 x + 1 = \sec^2 x$

42. What identity relates $\cot^2 x$ to $\csc^2 x$? $\cot^2 x + 1 = \csc^2 x$ → $\sec^2 x - 1$

43. How do you use your answers to the first two questions to integrate $\tan^2 x$ and $\cot^2 x$? → $\csc^2 x - 1$

44. What are the three possibilities for evaluating an integral expression involving powers of tan and sec?

- $u = \tan x$, $du = \sec^2 x\,dx$
- $u = \sec x$, $du = \sec x \tan x\,dx$
- Switch to sin & cos

45. How do you decide which to use? whichever allows conversion of all other terms

Numerical Methods

46. State the formula for finding the area of a trapezoid. $\frac{h}{2}(b_1 + b_2)$

47. In the Trapezoidal Rule, the partition is regular. What is $\Delta x$? $\frac{b-a}{n}$

48. What is the Trapezoidal Rule? $\frac{1}{2} \cdot \frac{b-a}{n}\left[y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-2} + 2y_{n-1} + y_n\right]$

49. How do you create a trapezoidal approximation for a non regular partition? consider each trap

50. To increase the accuracy of a trapezoidal approximation, what must be done?

increase n