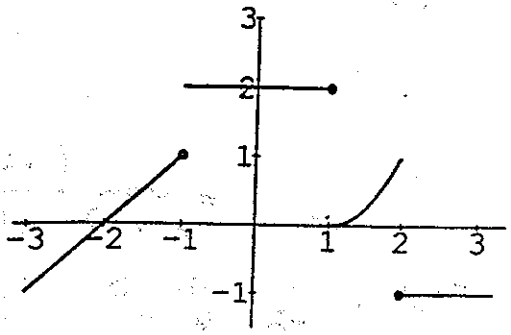
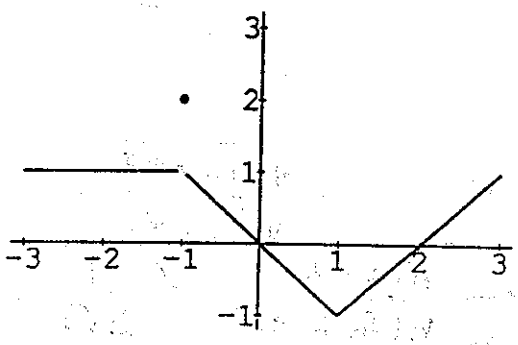
11.

The graphs of the functions f and g are given below.



graph of f



graph of g

Determine whether the following limits exist. If they do, then find the limit.

a. $\lim_{x\to -1} f(x)$ DNE $\lim_{x\to -1^+} f(x) = 2$, $\lim_{x\to -1^-} f(x) = 1$

b. $\lim_{x\to 1} f(x)$ DNE $R \neq L$ 0 2

c. $\lim_{x\to -1} g(x)$ 1 $R \neq L$ 3 2

d. $\lim_{x\to 1} g(x) = -1$

e. $\lim_{x\to -1} f(x)+g(x)$ DNE

f. $\lim_{x\to 0} 2f(x)+3g(x) = 4$ $2\cdot 2+3\cdot 0$

g. $\lim_{x\to -1} f(x)g(x)$ DNE $R \neq L$ 2 1

h. $\lim_{x\to 2} f(x)g(x)$ 0 $R = L$ $-1\cdot 0 = 0$ $1\cdot 0 = 0$

i. $\lim_{x\to 0} \frac{f(x)}{g(x)}$ DNE $-\infty$ ∞

j. $\lim_{x\to 0} \frac{g(x)}{f(x)}$ 0

k. $\lim_{x\to -2} g(f(x))$ 0

l. $\lim_{x\to -1} f(g(x))$ DNE $R \neq L$ 0 2

12. Determine whether the following limits exist. Find the limits if they do exist. Show all work.

a. $\lim_{x\to 0} \frac{\sin x}{|x|}$ DNE $\lim_{x\to 0^+} \frac{\sin x}{x} = 1$, $\lim_{x\to 0^-} \frac{\sin x}{-x} = -1$

b. $\lim_{x\to 0} \frac{1-\cos^2(3x)}{x^2} = \lim_{x\to 0} \frac{\sin^2 3x}{x^2} = \lim_{x\to 0} \frac{\sin 3x}{x}\cdot\frac{\sin 3x}{x} = 3\cdot 3 = 9$

c. $\lim_{x\to 0} \cos\left(\frac{1-\cos x}{x}\right) = \cos 0 = 1$

$\lim_{x\to 0} \frac{1-\cos x}{x}\cdot\frac{1+\cos x}{1+\cos x}$

$\lim_{x\to 0} \frac{1-\cos^2 x}{x(1+\cos x)}$

$\lim_{x\to 0} \frac{\sin x}{x}\cdot\frac{\sin x}{1+\cos x} = 1\cdot\frac{0}{2} = 0$

$\lim_{x\to 0} \frac{\sin 3x}{x} = \lim_{t\to 0} \frac{\sin t}{\frac{t}{3}}$

$t = 3x$, $x\to 0$, $t\to 0$

$= \lim_{t\to 0} 3\cdot\frac{\sin t}{t} = 3$