AP Calculus

WS: Limits 1

Name________________________________

If possible, evaluate each of the following limits. Show all work. No calculators. Please circle your answers.

1. $\lim\limits_{x\to 7}|3x-11| = 10$

$|21-11| = |10|$

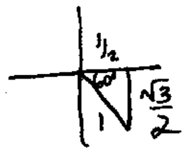
2. $\lim\limits_{x\to -3}\left(3x^2+7x-5\right) = 1$

$27-21-5$

3. $\lim\limits_{x\to \frac{\pi}{2}}\sin 2x = 0$

$\sin \pi$

4. $\lim\limits_{x\to 5}\tan\dfrac{\pi x}{3} = -\sqrt{3}$



5. $\lim\limits_{x\to \frac{1}{2}}[3x-1] = 0$

$\left[\frac{3}{2}-1\right] = \left[\frac{1}{2}\right]$

6. $\lim\limits_{x\to 5}\dfrac{x-5}{x^2-25} = \lim\limits_{x\to 5}\dfrac{x-5}{(x-5)(x+5)} = \dfrac{1}{10}$

7. $\lim\limits_{x\to 3^+}\dfrac{3-x}{|x-3|} = -1$

$\lim\limits_{x\to 3^+}\dfrac{3-x}{x-3}$ If $x>3$, $x-3$ is $+$

8. $\lim\limits_{x\to 4}\dfrac{\sqrt{x}-2}{x-4}\cdot\dfrac{\sqrt{x}+2}{\sqrt{x}+2} = \lim\limits_{x\to 4}\dfrac{x-4}{(x-4)(\sqrt{x}+2)} = \dfrac{1}{4}$

9. $\lim\limits_{x\to 0}\dfrac{x}{\sin 2x} = \lim\limits_{t\to 0}\dfrac{t/2}{\sin t}$

$t = 2x$

$x\to 0$

$t\to 0$

$= \lim\limits_{t\to 0}\dfrac{1}{2}\cdot\dfrac{t}{\sin t} = \dfrac{1}{2}$

10. $\lim\limits_{x\to 1}\dfrac{x^3-1}{x-1} = \lim\limits_{x\to 1}\dfrac{(x-1)(x^2+x+1)}{x-1} = 3$

11. $\lim\limits_{x\to 1^-}\dfrac{2x-2}{|x-1|}$

$\lim\limits_{x\to 1^-}\dfrac{2(x-1)}{1-x} = -2$

12. $\lim\limits_{x\to -4}\dfrac{2x-1}{(x+4)^2} = -\infty$

$\left(\dfrac{-9}{0^+}\right)$

13. $\lim\limits_{x\to 0}(x\cdot\csc x)$

$\lim\limits_{x\to 0} x\cdot\dfrac{1}{\sin x} = \lim\limits_{x\to 0}\dfrac{x}{\sin x} = 1$

14. $\lim\limits_{x\to 2^+}\dfrac{4-x^2}{|2-x|} = \lim\limits_{x\to 2^+}\dfrac{(2-x)(2+x)}{x-2}$

$= -1(4)$

$= -4$

15. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{1-\sin x}{\cos x}\cdot\dfrac{1+\sin x}{1+\sin x}$

$\lim\limits_{x\to \pi/2}\dfrac{1-\sin^2 x}{\cos x(1+\sin x)}$

$\lim\limits_{x\to \pi/2}\dfrac{\cos x}{1+\sin x} = \dfrac{0}{1+1} = 0$

AP Calculus
WS: Limits 2

Show any work you must do to evaluate each limit. No calculators.

1. $\lim_{x\to 3}\sqrt{\frac{x^2-9}{x^2-3x}} = \lim_{x\to 3}\sqrt{\frac{(x+3)\cancel{(x-3)}}{x\cancel{(x-3)}}} = \sqrt{2}$

2. $\lim_{x\to 0}\frac{2-\sqrt{4+x}}{x}\cdot\frac{2+\sqrt{4+x}}{2+\sqrt{4+x}} = \lim_{x\to 0}\frac{4-4-x}{x(2+\sqrt{4+x})} = -\frac{1}{4}$

3. $\lim_{x\to 0^-}\frac{x^2-2x}{|x|} = \lim_{x\to 0^-}\frac{x(x-2)}{-x} = -1(-2) = 2$

4. $\lim_{x\to 2^+}\frac{x+1}{2x-x^2} = \lim_{x\to 2^+}\frac{x+1}{x(2-x)} = -\infty$ $\left(\frac{2}{2\cdot 0^-}\right)$

5. $\lim_{x\to 0}\frac{\tan^2 x}{x} = \lim_{x\to 0}\frac{\sin^2 x}{x\cos^2 x} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin x}{\cos^2 x} = 1\cdot 0 = 0$

6. $\lim_{x\to 0}\frac{2\sin 4x}{3x} = \frac{2}{3}\lim_{x\to 0}\frac{\sin 4x}{x} = \lim_{t\to 0}\frac{\sin t}{\frac{t}{4}} = 4\cdot\lim_{t\to 0}\frac{\sin t}{t}$

$t=4x$, $x\to 0$, $t\to 0$

$= 4$

7. $\lim_{x\to 0}\frac{1-\cos 5x}{x}\cdot\frac{1+\cos 5x}{1+\cos 5x} = 0$

$\lim_{x\to 0}\frac{1-\cos^2 5x}{x(1+\cos 5x)} = \lim_{x\to 0}\frac{\sin 5x}{x}\cdot\frac{\sin 5x}{1+\cos 5x} = 5\cdot\frac{0}{2}$ (+)

8. $\lim_{x\to 0} x\cos\frac{1}{x} = 0$ squeeze

$-1 \le \cos\frac{1}{x} \le 1$

$x<0$: $x \le x\cos\frac{1}{x} \le -x$; $\lim_{x\to 0^-} x = 0$; $\lim_{x\to 0^-} -x = 0$

$x>0$: $-x \le x\cos\frac{1}{x} \le x$; $\lim_{x\to 0^+} -x = 0$; $\lim_{x\to 0^+} x = 0$

9. $\lim_{x\to\infty} x\sin\frac{1}{x}$ $\quad \lim_{t\to 0^+}\frac{1}{t}\cdot\sin t = 1$

$t=\frac{1}{x}$ $\quad x\to\infty$, $t\to 0^+$

10. $\lim_{x\to 1}[x]$ DNE

$\lim_{x\to 1^+}[x] = 1$; $\lim_{x\to 1^-}[x] = 0$

* 11. $\lim_{h\to 0}\frac{\cos\left(\frac{\pi}{2}+h\right)-\cos\frac{\pi}{2}}{h} = -1$

12. $\lim_{x\to 0}\sin\frac{1}{x}$ DNE

$t=\frac{1}{x}$

$x\to 0^+$ $\quad x\to 0^-$

$t\to\infty$ $\quad t\to -\infty$

$\lim_{t\to\infty}\sin t$ DNE

$\lim_{t\to -\infty}\sin t$ DNE

13. $\lim_{x\to\infty}\frac{1}{2-x} = 0^{(-)}$

14. $\lim_{x\to 2^-}\frac{1}{2-x} = \infty$ $\left(\frac{1}{0^+}\right)$

15. $\lim_{x\to 1^-}\frac{2x+2}{x^2-1} = -\infty$

$\frac{2\cancel{(x+1)}}{\cancel{(x+1)}(x-1)}$ $\left(\frac{2}{0^-}\right)$

16. $\lim_{x\to\infty}\frac{1-x^3}{1-x} = \infty$

x^2

17. $\lim_{x\to -\infty}\frac{4-5x}{|x|} = \lim_{x\to -\infty}\frac{4-5x}{-x} = 5$

18. $\lim_{x\to\infty}\frac{4-5x}{|x|} = \lim_{x\to\infty}\frac{4-5x}{x} = -5$

* 11. $\lim_{h\to 0}\frac{\cos\frac{\pi}{2}\cos h-\sin\frac{\pi}{2}\sin h - 0}{h}$

$= \lim_{h\to 0}\frac{-1\cdot\sin h}{h} = -1$

(+) $\lim_{x\to 0}\frac{\sin 5x}{x} = \lim_{t\to 0}\frac{\sin t}{\frac{t}{5}}$

$t=5x$ $\quad = 5\cdot\lim_{t\to 0}\frac{\sin t}{t}$

$x\to 0$

$t\to 0$ $\quad = 5$

AP Calculus
WS: Limits 3

Show any work you must do to evaluate each limit. No calculators.

1. $\lim_{r\to 1}\sqrt{\frac{8r+1}{r+3}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

2. $\lim_{s\to\infty}\frac{3s^2-8s-16}{2s^2-9s+4} = \frac{3}{2}$

3. $\lim_{h\to 0}\frac{\sqrt{h+2}-\sqrt{2}}{h}\cdot\frac{\sqrt{h+2}+\sqrt{2}}{\sqrt{h+2}+\sqrt{2}}$

$\lim_{h\to 0}\frac{h+2-2}{h(\sqrt{h+2}+\sqrt{2})} = \frac{1}{2\sqrt{2}}$

4. $\lim_{x\to -2}\frac{x^3-x^2-x+10}{x^2+3x+2} = \lim_{x\to -2}\frac{(x+2)(x^2-3x+5)}{(x+2)(x+1)}$

$$\begin{array}{r|rrr|r} -2 & 1 & -1 & -1 & 10 \\ & & -2 & 6 & -10 \\ \hline & 1 & -3 & 5 & 0 \end{array}$$

$= \frac{15}{-1} = -15$

5. $\lim_{t\to 2^-}\frac{-t^2+t+2}{t^2-4t+4}$

$\left(\frac{-3}{0^-}\right)$

$\lim_{t\to 2^-}\frac{-t^2+t+2}{(t-2)^2} = \lim_{t\to 2^-}\frac{-\cancel{(t-2)}(t+1)}{(t-2)^{\cancel{2}}} = \infty$

6. $\lim_{s\to 2^-}\left(\frac{1}{s-2}-\frac{3}{s^2-4}\right) = \lim_{s\to 2^-}\frac{s+2-3}{(s+2)(s-2)}$

$= \lim_{s\to 2^-}\frac{s-1}{(s+2)(s-2)} = -\infty$

$\left(\frac{1}{4\cdot 0^-}\right)$

7. $\lim_{x\to 2^+}\frac{x^4-16}{|2-x|} = 32$

$\lim_{x\to 2^+}\frac{(x^2-4)(x^2+4)}{x-2} = \lim_{x\to 2}(x+2)(x^2+4)$

8. $\lim_{x\to 0}\frac{x^2}{\sin^2(4x)} = \lim_{x\to 0}\frac{x}{\sin 4x}\cdot\frac{x}{\sin 4x}$ (+)

$= \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}$

9. $\lim_{x\to\infty}\left(x\cdot\cos\frac{1}{x}\right)$

$t=\frac{1}{x}$
$x\to\infty$
$t\to 0^+$

$\lim_{t\to 0^+}\frac{1}{t}\cdot\cos t = \infty$

$\left(\frac{1}{0^+}\right)$

10. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ if $f(x) = x^2$

$\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}$

$\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}}$

$= 2x+0 = 2x$

(+) $\lim_{x\to 0}\frac{x}{\sin 4x} = \lim_{t\to 0}\frac{\frac{t}{4}}{\sin t}$

$t=4x$
$x\to 0$
$t\to 0$

$= \frac{1}{4}\lim_{t\to 0}\frac{t}{\sin t}$

$= \frac{1}{4}$