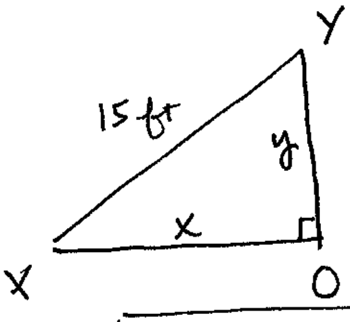
EXAM REVIEW
SOLUTIONS/RUBRICS

Unless otherwise noted,
1 Box = 1 point.
Equivalent expressions are accepted, but if you are unsure if your response would get credit, just ask.

AB / BC 1



$\boxed{\frac{dx}{dt} = \frac{1}{2} \text{ ft/sec}}$

a) $\left.\frac{dy}{dt}\right|_{\substack{x=9 \ \& \\ y=12}} = ?$

$x^2 + y^2 = 15^2$

$\boxed{2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0}$

$18(1/2) + 24\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -9/24$

$\boxed{\frac{dy}{dt} = -3/8 \text{ ft/sec}}$

$\boxed{\text{Y moves down wall at } 3/8 \text{ ft/sec}}$

b) $\left.\frac{d}{dt}\left[\frac{1}{2}xy\right]\right|_{\substack{x=9 \\ y=12}}$ $\qquad A = \frac{1}{2}xy$

$\frac{dA}{dt} = \boxed{\frac{1}{2}}\left[\boxed{y \cdot \frac{dx}{dt}} + \boxed{x \cdot \frac{dy}{dt}}\right]$ should be answer to a

$= \frac{1}{2}\left(12 \cdot 1/2 + 9 \cdot -3/8\right)$

$= \frac{1}{2}\left(6 - \frac{27}{8}\right)$

$= 3 - \frac{27}{16}$

$= \frac{48}{16} - \frac{27}{16} = \boxed{\frac{21}{16}}$

Area $\boxed{\text{increases}}$ at $\frac{21}{16}$ ft^2/sec.

AB 2 / BC 2

a) $\boxed{f' \text{ changes } + \text{ to } -}$ for f to have a rel. max

$\Rightarrow \boxed{x = 2}$

b)

x	$f(x)$
-1	-1
4	1

> given

no x values produce a rel. min since

$\boxed{f' \text{ never changes neg to pos.}}$

$\Rightarrow \boxed{x = -1 \text{ produces abs. min}}$

c) f is concave down when $\boxed{f' \text{ decreases}}$

$\Rightarrow \boxed{(-1, 0) \cup (1, 3)}$

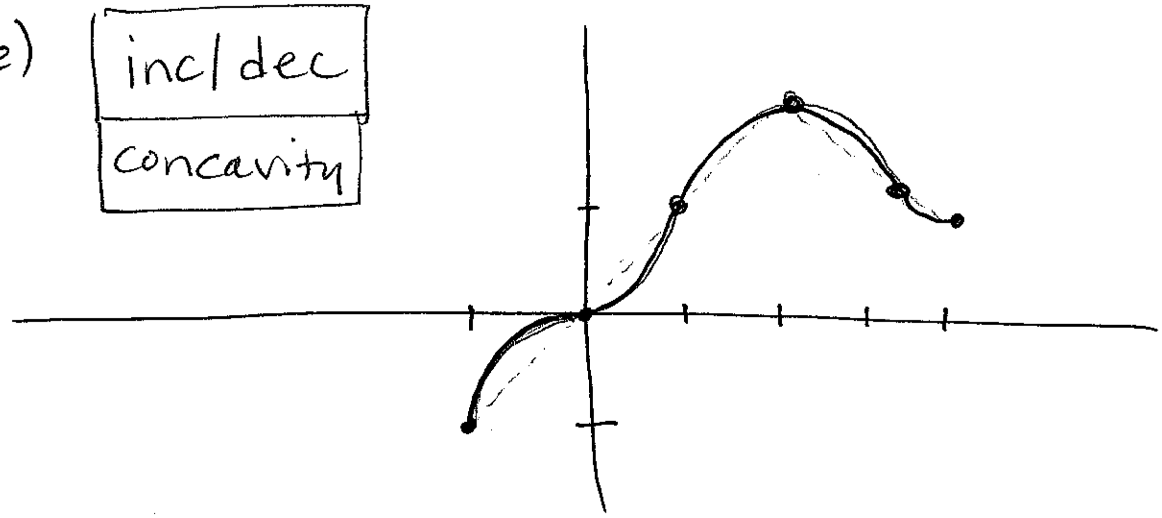
d) f has an inflection point when

$\boxed{f' \text{ has a rel. max or min}}$

$\Rightarrow \boxed{x = 0, 1, 3}$

e) $\boxed{\text{inc/dec}}$

$\boxed{\text{concavity}}$



f inc on $(-1, 2)$ since $f' > 0$

f dec on $(2, 4)$ since $f' < 0$

f cc up on $(0,1) \cup (3,4)$ since f' inc

AB/BC 3

a) f is even ⇒ no odd terms ⇒ $\boxed{a=0}$

$4-c=0$ to make $x=2$ V.A. ⇒ $\boxed{c=4}$

$$f'(x)=\frac{d}{dx}\left[\frac{b}{x^2-4}\right]=\frac{d}{dx}\left[b(x^2-4)^{-1}\right]$$

$$\boxed{f'(x)=b\cdot -1(x^2-4)^{-2}\cdot 2x}$$

$$f'(1)=b\cdot\frac{-1}{9}\cdot 2=-2$$

$$\boxed{b=9}$$

b) $$\lim_{x\to\infty}\frac{9}{x^2-4}=0 \qquad \lim_{x\to-\infty}\frac{9}{x^2-4}=0$$

(Must have both for point)

$\boxed{y=0 \text{ is horiz. asymp.}}$

$$\lim_{x\to-2^+}\frac{9}{x^2-4}=-\infty \quad \left(\frac{9}{0^-}\right)$$

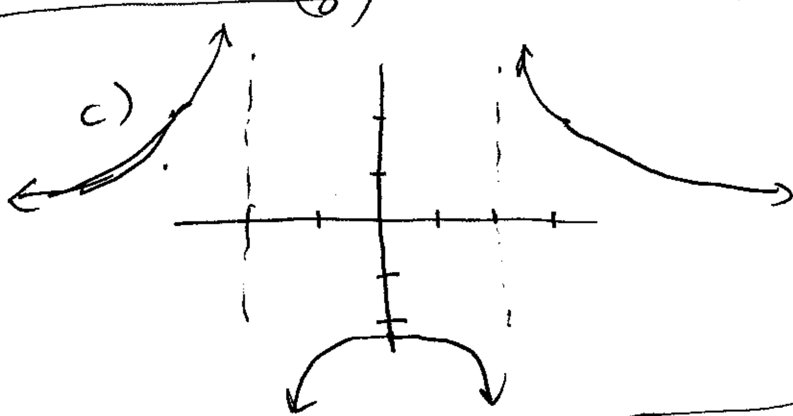
$$\lim_{x\to-2^-}\frac{9}{x^2-4}=\infty \quad \left(\frac{9}{0^+}\right)$$

$x=2$ is vertical asymp.

$x=-2$ is vertical asymp.

(or use even info)

c)



AB4 / BC4

a. $(k, 12-k^2)$ $(4,0)$ are on tangent at $x=k$

$f(x) = 12 - x^2$

$f'(x) = -2x \rightarrow \boxed{f'(k) = -2k}$

$$\boxed{-2k = \frac{12-k^2-0}{k-4}}$$ $(m_{tan} = m_{\text{points on tan}})$

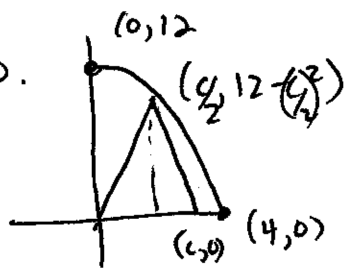
$-2k^2 + 8k = 12 - k^2$

$0 = k^2 - 8k + 12 = \boxed{(k-6)(k-2)}$

$k = 2, 6$

$(2,8)$ or $(6, \cancel{-24})$ $f(x) \geq 0$ given

$\Rightarrow \boxed{k=2}$

b. 

$0 < c < 4$

max $\boxed{A = \frac{1}{2} \cdot c \cdot \left(12 - \frac{c^2}{4}\right)}$

$A = 6c - \frac{c^3}{8}$

$\boxed{A' = 6 - \frac{3c^2}{8} = 0}$ if

$6 = \frac{3c^2}{8}$

$16 = c^2$

$\pm 4 = c \rightarrow \boxed{c = 4}$

or A' # line
+ to - $\rightarrow$
for this point
and verbal

$\boxed{A'' = -\frac{6c}{8} \quad A''(4) < 0}$

$\hookrightarrow$ $\boxed{A \text{ has horiz. tang. and is cc dn. at } 4 \Rightarrow \text{rel. max}}$

AB 5/BC5

a. $\boxed{\frac{d}{dx}\left[5x^3 + 40 = \int_c^x f(t)\,dt\right]}$

$$\boxed{15x^2 = f(x)}$$

$$5x^3 + 40 = \int_c^x 15t^2\,dt$$

$$5x^3 + 40 = \boxed{\frac{15t^3}{3}\Big|_c^x} = \frac{15x^3}{3} - \frac{15c^3}{3}$$

$$\boxed{40 = -\frac{15c^3}{3} = -5c^3}$$

$$-8 = c^3$$

$$\boxed{-2 = c}$$

b. $\boxed{F'(x) = -\sqrt{1+x^{16}}}$ 2 points function

2 points –

AB6 / BC6

a. $g(6) = 5 + \int_6^6 f(t)\,dt = 5 + 0 = \boxed{5}$

$\boxed{g'(x) = f(x)}$ ← whole box = 1

$g'(6) = f(6) = \boxed{3}$

$\boxed{g''(x) = f'(x)}$ ← whole box = 1

$g''(6) = f'(6) = \boxed{0}$ since f has horiz. tang. at 6

b. g dec if $g' = \boxed{f \text{ is negative}}$

$\Rightarrow \boxed{(-3,0) \cup (12,15)}$

c. g is cc down if $g' = \boxed{f \text{ dec.}}$

$\Rightarrow \boxed{(6,15)}$

d. $\Delta t = 3 \Rightarrow h = 3$

$\cancel{\frac{1}{2} \cdot \frac{3}{2}\left[(-3+0)+(0+1)+(1+3)+(3+1)+(-1+0)\right]}$

$\cancel{\frac{3}{2} \cdot \frac{3}{8}(}$

whole box

$\boxed{\frac{1}{2} \cdot 3} \left[(-1+0)+(0+1)+(1+3)+(3+1)+(1+0)+(0+1)\right]$

$\frac{3}{2}(8) = \boxed{12}$

AB 7/BC 7

a. $g(3) = \int_2^3 f(t)\,dt = \frac{1}{2}\cdot 1\cdot(4+2) = \boxed{3}$

$g'(x) = f(x) \Rightarrow g'(3) = f(3) = \boxed{2}$

$g''(x) = f'(x) \Rightarrow g''(3) = f'(3) = \boxed{-2}$ (slope of that section)

b. $\dfrac{g(3)-g(0)}{3-0} = \dfrac{3 \text{ (from a)} - (-4) \text{ (see below)}}{3} = \boxed{\dfrac{7}{3}}$

$g(0) = \int_2^0 f(t)\,dt = -\frac{1}{2}(2)(4) = \boxed{-4}$

c. $\boxed{\text{Two:}}$ $\boxed{y = 7/3 \text{ intersects } f(x) \text{ at two places}}$ on $[0,3]$

d. $g''(x)$ changes signs $\Rightarrow$ $f'(x)$ changes signs, so $\boxed{f \text{ changes inc to dec or dec to inc}}$ $\Rightarrow$ $\boxed{x = 2, 5}$

BC 8

a) $\boxed{\int_0^9 \boxed{(6-2\sqrt{x})}\,dx}$ or $\boxed{\int_0^6 \boxed{\frac{y^2}{4}}\,dy}$

$6x - 2\cdot\frac{2}{3}x^{3/2}\Big|_0^9$ $\boxed{\text{1pt}}$ $\qquad \frac{y^3}{12}\Big|_0^6$

$54 \; \cancel{36} - \frac{4}{3}\cdot 27$

$54 - 36$

18 $\qquad \boxed{\text{1 pt}}$

$\frac{\cancel{6}\cdot 6\cdot \cancel{6}\,3}{\cancel{12}\,2} = 18$

b) Washer: $O = 7-2\sqrt{x}$

$I = 1$

$h = dx$

$\boxed{\pi\int_0^9}\boxed{[(7-2\sqrt{x})^2 \boxed{-} 1]}\,dx$

OR

Shell: $r = 7-y$

$h = y^2/4$

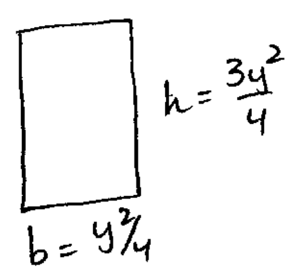
$w = dy$

$\underbrace{\boxed{2\pi}\boxed{\int_0^6}}_{\text{1 pt}}\boxed{(7-y)}\boxed{\left(\frac{y^2}{4}\right)}\,dy$

c) $B = \frac{3y^2}{16}$

$h = dy$

$\boxed{\int_0^6}\boxed{\frac{3}{16}y^2}\,dy$



AB 8/ BC 9

a. $6y^2\frac{dy}{dx} + 12xy + 6x^2\frac{dy}{dx} - 24x + 6\frac{dy}{dx} = 0$ 1 pt non $\frac{dy}{dx}$

1 pt dy/dx

$$\boxed{\frac{dy}{dx}\left(6y^2 + \overset{6x^2}{\cancel{12x}}y + 6\right) = 24x - 12xy}$$

$$\frac{dy}{dx} = \frac{12x(2-y)}{6(x^2+y^2+1)}$$

$$\frac{dy}{dx} = \frac{2x(2-y)}{x^2+y^2+1} = \boxed{\frac{4x-2xy}{x^2+y^2+1}}$$

b. $m=0 \Rightarrow \boxed{4x - 2yx = 0}$

$2x(2-y) = 0$

$x = 0$ or $y = 2$

$\boxed{2y^3 + 0 - 0 + 6y = 1}$

$2y^3 + 6y = 1$

$\boxed{y = 0.165}$

(intersect on calculator – no need to write this)

$16 + 12x^2 - 12x^2 + 12 = 1$

$\varnothing$

c. $P(x,y)$ to $(0,0)$ $m = -1$

$\frac{y}{x} = -1 \Rightarrow \boxed{y = -x}$

and $\boxed{2y^3 + 6x^2y - 12x^2 + 6y = 1}$

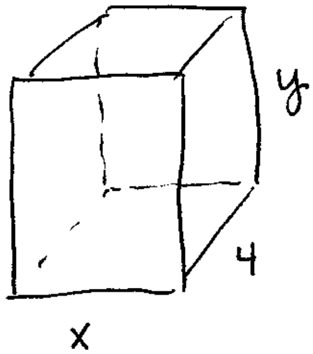
$-2x^3 + -6x^3 - 12x^2 - 6x = 1$

$-8x^3 - 12x^2 - 6x = 1$

$0 = 8x^3 + 12x^2 + 6x + 1$

$\boxed{x = -\frac{1}{2} \Rightarrow y = -(-\frac{1}{2}) = \frac{1}{2}}$

AB9/BC 10



$4xy = 36$

$xy = 9$

$\boxed{y = \frac{9}{x}}$

$$\min C = \boxed{10(4x)} + \boxed{5(4y + xy + xy + 4y)}$$

$$C = 40x + 40y + 10xy$$

$$\boxed{C = 40x + \frac{360}{x} + 90}$$

2pts → $\boxed{C' \text{ changes } - \text{ to } +}$ at $\boxed{x = 3}$

$\Rightarrow \boxed{C(3) \text{ is min}}$

min $\boxed{C = \$330}$

AB 10 / BC 11

Note: $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$

a) $P'(9) = \boxed{-.646}$

$\Rightarrow$ oil is decreasing at $t=9$ since $P'(9) < 0$

b) $P(t)$ has min when $\boxed{P'(t) \text{ changes } - \text{ to } +}$

when $\boxed{t = 30.173 \text{ or } 30.174}$

c) $\int_0^{30.173\ldots} P'(t)\,dt = P(30.173\ldots) - P(0)$

$-14.895\ldots = P(30.173\ldots) - 50$

$\boxed{35.104 = P(30.173)}$

Since the minimum is under 40 gallons,

$\boxed{\text{yes.}}$

d) $P(0) = 50$; $P'(0) = -2$

$\Rightarrow \boxed{y - 50 = -2(t - 0)}$

$40 - 50 = -2t$

$-10 = -2t$

$5 = t$

$\boxed{\text{5 days}}$

BC12

a) $\boxed{\int_0^2} \boxed{(6 - 4\ln(3-x))}\, dx = \boxed{6.816 \text{ or } 6.817}$

b) Washer (shell requires solving for x)

Outer = $8 - 4\ln(3-x)$

Inner = 2

$\boxed{\pi \int_0^2} \left(\boxed{(8-4\ln(3-x))^2} \boxed{-4}\right) dx = \boxed{168.179 \text{ or } 168.180}$

c) $\int_0^2 \boxed{(6-4\ln(3-x))^2}\, dx = \boxed{26.266 \text{ or } 26.267}$

AB11/BC13

a) $\boxed{\int_0^{12} H(t)\,dt} = \boxed{70.570 \text{ or } 70.571 \text{ gallons}}$

b) $\boxed{H(6) - R(6) = -2.924}$

$\Rightarrow$ falling since rate of change in amount of oil is negative at $t=6$

c) $\boxed{\int_0^{12} (H(t) - R(t))\,dt = A(12) - A(0)}$; $A(t)$ = Amount oil at time t

$-2.974\ldots = A(12) - 125$

$\boxed{122.025 \text{ or } 122.026 \text{ gallons}} = A(12)$

d) min $A(t)$ when $\boxed{A'(t) \text{ changes } - \text{ to } +}$ or at an $\boxed{\text{endpoint}}$

$A'(t) = H(t) - R(t)$ changes $-$ to $+$ at

$t = 11.318\ldots$

$A(0) = 125$

$A(12) = 122.026$

$A(11) - A(0) = \int_0^{11} (H(t) - R(t))\,dt$

$A(11) = 125 + -4.26\ldots$

$= 120.738$

$\Rightarrow \boxed{11.318 \text{ hours}}$

AB 12 / BC 14

a) $a(t) > 0$ when $\boxed{v(t) \text{ inc}} \Rightarrow \boxed{(0, 35) \cup (45, 50)}$

b) $$\boxed{\frac{v(50) - v(0)}{50 - 0} = \frac{72}{50} = 1.44 \text{ ft/sec}^2}$$

c) slope $t = 35$ to $t = 40$: $\frac{75 - 81}{5} = -1.2 \text{ ft/sec}^2$

or slope $t = 40$ to $t = 45$: $\frac{60 - 75}{5} = -3 \text{ ft/sec}^2$ 2 - 1 slope, 1 calcul.

or slope $t = 35$ to $t = 45$: $\frac{60 - 81}{10} = -2.1 \text{ ft/sec}^2$

d) $\Delta t = 10$ sec

$\boxed{10}\boxed{[12 + 30 + 70 + 81 + 60]} = \boxed{2530 \text{ ft}}$

estimates $\boxed{\text{distance traveled in 50 sec.}}$