BC Calculus
Rubrics
FR 7 & 8

Question1

$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$

Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \int_P^Q \left(e^{2x-x^2} - 2\right) dx = 0.514$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(b) $e^{2x-x^2} = 1$ when $x = 0, 2$

Area of $S = \int_0^2 \left(e^{2x-x^2} - 1\right)dx - \text{Area of } R$

$= 2.06016 - \text{Area of } R = 1.546$

OR

$\int_0^P \left(e^{2x-x^2} - 1\right) dx + (Q-P)\cdot 1 + \int_Q^2 \left(e^{2x-x^2} - 1\right) dx$

$= 0.219064 + 1.107886 + 0.219064 = 1.546$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(c) Volume $= \pi \int_P^Q \left(\left(e^{2x-x^2} - 1\right)^2 - (2-1)^2\right) dx$

$3: \begin{cases} 2: \text{integrand} \\ 1: \text{constant and limits} \end{cases}$

Question 2

(a) Speed $= \sqrt{x'(4)^2 + y'(4)^2} = 2.912$

$1: \text{speed at } t = 4$

(b) Distance $= \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 6.423$

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(c) $x(4) = x(0) + \int_0^4 x'(t)\,dt$

$= -3 + 2.10794 = -0.892$

$3: \begin{cases} 2: \begin{cases} 1: \text{integrand} \\ 1: \text{uses } x(0) = -3 \end{cases} \\ 1: \text{answer} \end{cases}$

(d) The slope is 2, so $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2$, or $\ln\left(t^2+1\right) = 2\arctan\left(\frac{t}{1+t}\right)$.

Since $t > 0$, $t = 1.35766$. At this time, the acceleration is $\langle x''(t), y''(t)\rangle\big|_{t=1.35766} = \langle 0.135, 0.955\rangle$.

$3: \begin{cases} 1: \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2 \\ 1: t\text{-value} \\ 1: \text{values for } x'' \text{ and } y'' \end{cases}$

Question 3

(a) $W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285$ or -0.286

When $v = 20$ mph, the wind chill is decreasing at $0.286\ °\text{F}/\text{mph}$.

$$2: \begin{cases} 1: \text{value} \\ 1: \text{explanation} \end{cases}$$

(b) The average rate of change of W over the interval $5 \le v \le 60$ is $\dfrac{W(60) - W(5)}{60 - 5} = -0.253$ or -0.254.

$W'(v) = \dfrac{W(60) - W(5)}{60 - 5}$ when $v = 23.011$.

$$3: \begin{cases} 1: \text{average rate of change} \\ 1: W'(v) = \text{average rate of change} \\ 1: \text{value of } v \end{cases}$$

(c) $\left.\dfrac{dW}{dt}\right|_{t=3} = \left.\left(\dfrac{dW}{dv} \cdot \dfrac{dv}{dt}\right)\right|_{t=3} = W'(35) \cdot 5 = -0.892\ °\text{F}/\text{hr}$

OR

$W = 55.6 - 22.1(20 + 5t)^{0.16}$

$\left.\dfrac{dW}{dt}\right|_{t=3} = -0.892\ °\text{F}/\text{hr}$

$$3: \begin{cases} 1: \dfrac{dv}{dt} = 5 \\ 1: \text{uses } v(3) = 35, \\ \quad \text{or} \\ \quad \text{uses } v(t) = 20 + 5t \\ 1: \text{answer} \end{cases}$$

Units of $°\text{F}/\text{mph}$ in (a) and $°\text{F}/\text{hr}$ in (c)

1 : units in (a) and (c)

Question 4

(a) $f'(x) = 0$ at $x = -3, 1, 4$

f' changes from positive to negative at -3 and 4.

Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

$$2: \begin{cases} 1: x\text{-values} \\ 1: \text{justification} \end{cases}$$

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4,\ -1,$ and 2. Thus, the graph of f has points of inflection when $x = -4,\ -1,$ and 2.

$$2: \begin{cases} 1: x\text{-values} \\ 1: \text{justification} \end{cases}$$

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

$$2: \begin{cases} 1: \text{intervals} \\ 1: \text{explanation} \end{cases}$$

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

$f(-5) = 3 + \int_1^{-5} f'(x)\,dx = 3 - \frac{\pi}{2} + 2\pi > 3$

$f(1) = 3$

$f(5) = 3 + \int_1^{5} f'(x)\,dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

$$3: \begin{cases} 1: \text{identifies } x = 1 \text{ as a candidate} \\ 1: \text{considers endpoints} \\ 1: \text{value and explanation} \end{cases}$$

Question 5

(a) $\dfrac{d^2y}{dx^2} = 3 + 2\dfrac{dy}{dx} = 3 + 2(3x + 2y + 1) = 6x + 4y + 5$

$2 : \begin{cases} 1 : 3 + 2\dfrac{dy}{dx} \\ 1 : \text{answer} \end{cases}$

(b) If $y = mx + b + e^{rx}$ is a solution, then
$m + re^{rx} = 3x + 2\left(mx + b + e^{rx}\right) + 1.$

If $r \neq 0$: $m = 2b + 1,\ r = 2,\ 0 = 3 + 2m,$
so $m = -\dfrac{3}{2},\ r = 2,$ and $b = -\dfrac{5}{4}.$

OR

If $r = 0$: $m = 2b + 3,\ r = 0,\ 0 = 3 + 2m,$
so $m = -\dfrac{3}{2},\ r = 0,\ b = -\dfrac{9}{4}.$

$3 : \begin{cases} 1 : \dfrac{dy}{dx} = m + re^{rx} \\ 1 : \text{value for } r \\ 1 : \text{values for } m \text{ and } b \end{cases}$

(c) $f\left(\dfrac{1}{2}\right) \approx f(0) + f'(0)\cdot\dfrac{1}{2} = -2 + (-3)\cdot\dfrac{1}{2} = -\dfrac{7}{2}$

$f'\left(\dfrac{1}{2}\right) \approx 3\left(\dfrac{1}{2}\right) + 2\left(-\dfrac{7}{2}\right) + 1 = -\dfrac{9}{2}$

$f(1) \approx f\left(\dfrac{1}{2}\right) + f'\left(\dfrac{1}{2}\right)\cdot\dfrac{1}{2} = -\dfrac{7}{2} + \left(-\dfrac{9}{2}\right)\cdot\dfrac{1}{2} = -\dfrac{23}{4}$

$2 : \begin{cases} 1 : \text{Euler's method with 2 steps} \\ 1 : \text{Euler's approximation for } f(1) \end{cases}$

(d) $g'(0) = 3\cdot 0 + 2\cdot k + 1 = 2k + 1$
$g(1) \approx g(0) + g'(0)\cdot 1 = k + (2k + 1) = 3k + 1 = 0$
$k = -\dfrac{1}{3}$

$2 : \begin{cases} 1 : g(0) + g'(0)\cdot 1 \\ 1 : \text{value of } k \end{cases}$

Question 6

(a) $f(x) = 6\left[1 - \dfrac{x}{3} + \dfrac{x^2}{2!3^2} - \dfrac{x^3}{3!3^3} + \cdots + \dfrac{(-1)^n x^n}{n!3^n} + \cdots\right]$

$= 6 - 2x + \dfrac{x^2}{3} - \dfrac{x^3}{27} + \cdots + \dfrac{6(-1)^n x^n}{n!3^n} + \cdots$

$3 : \begin{cases} 1 : \text{two of } 6, -2x, \dfrac{x^2}{3}, -\dfrac{x^3}{27} \\ 1 : \text{remaining terms} \\ 1 : \text{general term} \end{cases}$

$\langle -1 \rangle$ missing factor of 6

(b) $g(0) = 0$ and $g'(x) = f(x)$, so

$g(x) = 6\left[x - \dfrac{x^2}{6} + \dfrac{x^3}{3!3^2} - \dfrac{x^4}{4!3^3} + \cdots + \dfrac{(-1)^n x^{n+1}}{(n+1)!3^n} + \cdots\right]$

$= 6x - x^2 + \dfrac{x^3}{9} - \dfrac{x^4}{4(27)} + \cdots + \dfrac{6(-1)^n x^{n+1}}{(n+1)!3^n} + \cdots$

$3 : \begin{cases} 1 : \text{two terms} \\ 1 : \text{remaining terms} \\ 1 : \text{general term} \end{cases}$

$\langle -1 \rangle$ missing factor of 6

(c) $f'(x) = -2e^{-x/3}$, so $h(x) = -2k\,e^{-ax/3}$

$h(x) = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots + \dfrac{x^n}{n!} + \cdots = e^x$

$-2k\,e^{-ax/3} = e^x$

$\dfrac{-a}{3} = 1$ and $-2k = 1$

$a = -3$ and $k = -\dfrac{1}{2}$

OR

$f'(x) = -2 + \dfrac{2}{3}x + \cdots$, so

$h(x) = kf'(ax) = -2k + \dfrac{2}{3}akx + \cdots$

$h(x) = 1 + x + \cdots$

$-2k = 1$ and $\dfrac{2}{3}ak = 1$

$k = -\dfrac{1}{2}$ and $a = -3$

$3 : \begin{cases} 1 : \text{computes } k\,f'(ax) \\ 1 : \text{recognizes } h(x) = e^x, \\ \quad\quad \text{or} \\ \quad \text{equates 2 series for } h(x) \\ 1 : \text{values for } a \text{ and } k \end{cases}$