BC Calculus
Rubrics FR 3 & 4

## Question 1

(a) $a(7.5) = v'(7.5) = \dfrac{v(8) - v(7)}{8 - 7} = -0.1 \text{ miles/minute}^2$

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{units} \end{cases}$

(b) $\int_0^{12} |v(t)|\, dt$ is the total distance, in miles, that Caren rode during the 12 minutes from $t = 0$ to $t = 12$.

$$\int_0^{12} |v(t)|\, dt = \int_0^{2} v(t)\, dt - \int_2^{4} v(t)\, dt + \int_4^{12} v(t)\, dt$$
$$= 0.2 + 0.2 + 1.4 = 1.8 \text{ miles}$$

$2 : \begin{cases} 1 : \text{meaning of integral} \\ 1 : \text{value of integral} \end{cases}$

(c) Caren turns around to go back home at time $t = 2$ minutes. This is the time at which her velocity changes from positive to negative.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(d) $\int_0^{12} w(t)\, dt = 1.6$; Larry lives 1.6 miles from school.

$\int_0^{12} v(t)\, dt = 1.4$; Caren lives 1.4 miles from school.

Therefore, Caren lives closer to school.

$3 : \begin{cases} 2 : \text{Larry's distance from school} \\ \quad 1 : \text{integral} \\ \quad 1 : \text{value} \\ 1 : \text{Caren's distance from school} \\ \quad \text{and conclusion} \end{cases}$

## Question 2

(a) $\int_0^{2} R(t)\, dt = 980$ people

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $R'(t) = 0$ when $t = 0$ and $t = 1.36296$

The maximum rate may occur at 0, $a = 1.36296$, or 2.

$R(0) = 0$

$R(a) = 854.527$

$R(2) = 120$

The maximum rate occurs when $t = 1.362$ or $1.363$.

$3 : \begin{cases} 1 : \text{considers } R'(t) = 0 \\ 1 : \text{interior critical point} \\ 1 : \text{answer and justification} \end{cases}$

(c) $w(2) - w(1) = \int_1^{2} w'(t)\, dt = \int_1^{2} (2 - t) R(t)\, dt = 387.5$

The total wait time for those who enter the auditorium after time $t = 1$ is 387.5 hours.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $\dfrac{1}{980} w(2) = \dfrac{1}{980} \int_0^{2} (2 - t) R(t)\, dt = 0.77551$

On average, a person waits 0.775 or 0.776 hour.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

## Question 3

(a) $\frac{dy}{dt} = 0$ only when $t = 0.36735$. Let $b = 0.36735$.

The maximum vertical distance from the water surface to the diver's shoulders is

$$y(b) = 11.4 + \int_0^b \frac{dy}{dt}\, dt = 12.061 \text{ meters.}$$

Alternatively, $y(t) = 11.4 + 3.6t - 4.9t^2$, so $y(b) = 12.061$ meters.

$3 : \begin{cases} 1 : \text{considers } \frac{dy}{dt} = 0 \\ 1 : \text{integral or } y(t) \\ 1 : \text{answer} \end{cases}$

(b) $y(A) = 11.4 + \int_0^A \frac{dy}{dt}\, dt = 11.4 + 3.6A - 4.9A^2 = 0$ when $A = 1.936$ seconds.

$2 : \begin{cases} 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^A \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 12.946$ meters

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) At time $A$, $\frac{dy}{dx} = \left.\frac{dy/dt}{dx/dt}\right|_{t=A} = -19.21913.$

The angle between the path of the diver and the water is $\tan^{-1}(19.21913) = 1.518$ or $1.519$.

$2 : \begin{cases} 1 : \frac{dy}{dx} \text{ at time } A \\ 1 : \text{answer} \end{cases}$

## Question 4

(a) $f\left(-\frac{1}{2}\right) = f(-1) + \left(\left.\frac{dy}{dx}\right|_{(-1,2)}\right) \cdot \Delta x$

$= 2 + 4 \cdot \frac{1}{2} = 4$

$f(0) \approx f\left(-\frac{1}{2}\right) + \left(\left.\frac{dy}{dx}\right|_{\left(-\frac{1}{2},4\right)}\right) \cdot \Delta x$

$\approx 4 + \frac{1}{2} \cdot \frac{1}{2} = \frac{17}{4}$

$2 : \begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{answer} \end{cases}$

(b) $P_2(x) = 2 + 4(x+1) - 6(x+1)^2$

1 : answer

(c) $\frac{dy}{dx} = x^2(6 - y)$

$\int \frac{1}{6-y}\, dy = \int x^2\, dx$

$-\ln|6 - y| = \frac{1}{3}x^3 + C$

$-\ln 4 = -\frac{1}{3} + C$

$C = \frac{1}{3} - \ln 4$

$\ln|6 - y| = -\frac{1}{3}x^3 - \left(\frac{1}{3} - \ln 4\right)$

$|6 - y| = 4e^{-\frac{1}{3}\left(x^3+1\right)}$

$y = 6 - 4e^{-\frac{1}{3}\left(x^3+1\right)}$

$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

## Question 5

(a) $f'(4) \approx \dfrac{f(5) - f(3)}{5 - 3} = -3$

1 : answer

(b) $\int_2^{13} (3 - 5f'(x))\,dx = \int_2^{13} 3\,dx - 5\int_2^{13} f'(x)\,dx$
$= 3(13 - 2) - 5(f(13) - f(2)) = 8$

$2 : \begin{cases} 1 : \text{uses Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

(c) $\int_2^{13} f(x)\,dx \approx f(2)(3 - 2) + f(3)(5 - 3)$
$+ f(5)(8 - 5) + f(8)(13 - 8) = 18$

$2 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{answer} \end{cases}$

(d) An equation for the tangent line is $y = -2 + 3(x - 5)$.
Since $f''(x) < 0$ for all $x$ in the interval $5 \le x \le 8$, the line tangent to the graph of $y = f(x)$ at $x = 5$ lies above the graph for all $x$ in the interval $5 < x \le 8$.
Therefore, $f(7) \le -2 + 3 \cdot 2 = 4$.

An equation for the secant line is $y = -2 + \frac{5}{3}(x - 5)$.
Since $f''(x) < 0$ for all $x$ in the interval $5 \le x \le 8$, the secant line connecting $(5, f(5))$ and $(8, f(8))$ lies below the graph of $y = f(x)$ for all $x$ in the interval $5 < x < 8$.
Therefore, $f(7) \ge -2 + \frac{5}{3} \cdot 2 = \frac{4}{3}$.

$4 : \begin{cases} 1 : \text{tangent line} \\ 1 : \text{shows } f(7) \le 4 \\ 1 : \text{secant line} \\ 1 : \text{shows } f(7) \ge \frac{4}{3} \end{cases}$

## Question 6

(a) $1 + (x-1)^2 + \dfrac{(x-1)^4}{2} + \dfrac{(x-1)^6}{6} + \cdots + \dfrac{(x-1)^{2n}}{n!} + \cdots$

$2 : \begin{cases} 1 : \text{first four terms} \\ 1 : \text{general term} \end{cases}$

(b) $1 + \dfrac{(x-1)^2}{2} + \dfrac{(x-1)^4}{6} + \dfrac{(x-1)^6}{24} + \cdots + \dfrac{(x-1)^{2n}}{(n+1)!} + \cdots$

$2 : \begin{cases} 1 : \text{first four terms} \\ 1 : \text{general term} \end{cases}$

(c) $\lim\limits_{n\to\infty} \left| \dfrac{\frac{(x-1)^{2n+2}}{(n+2)!}}{\frac{(x-1)^{2n}}{(n+1)!}} \right| = \lim\limits_{n\to\infty} \dfrac{(n+1)!}{(n+2)!}(x-1)^2 = \lim\limits_{n\to\infty} \dfrac{(x-1)^2}{n+2} = 0$

Therefore, the interval of convergence is $(-\infty, \infty)$.

$3 : \begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{answer} \end{cases}$

(d) $f''(x) = 1 + \dfrac{4 \cdot 3}{6}(x-1)^2 + \dfrac{6 \cdot 5}{24}(x-1)^4 + \cdots$
$+ \dfrac{2n(2n-1)}{(n+1)!}(x-1)^{2n-2} + \cdots$

Since every term of this series is nonnegative, $f''(x) \ge 0$ for all $x$.
Therefore, the graph of $f$ has no points of inflection.

$2 : \begin{cases} 1 : f''(x) \\ 1 : \text{answer} \end{cases}$