

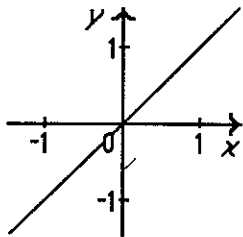
Answer Key: Concepts Worksheets

Chapter 1

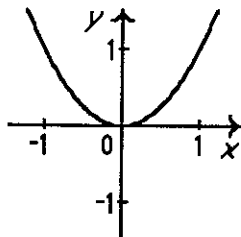
Worksheet 1

Section I

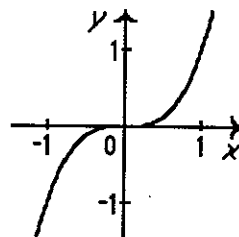
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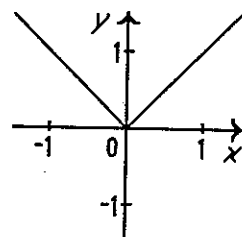
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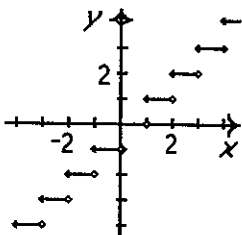
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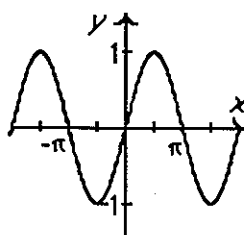
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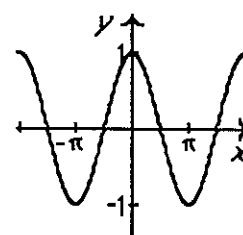
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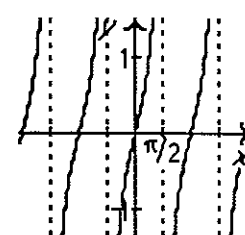
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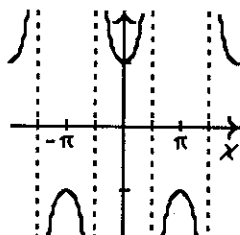
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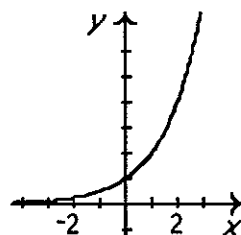
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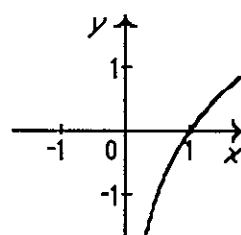
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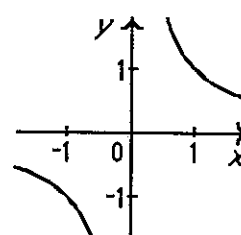
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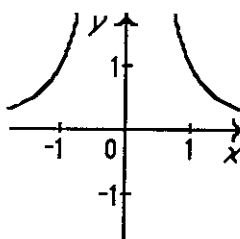
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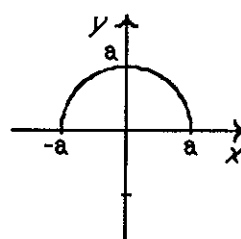
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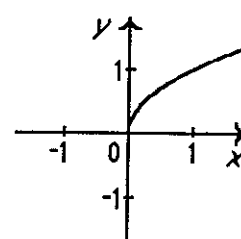
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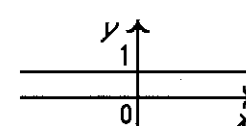
14.



15.



16.



Note: The naked eye cannot see the holes above each line. This is a function.

Section II (answers 1–16 on chart)

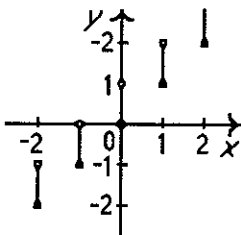
Function	Domain	Range $y = f(x)$	Roots (Find x when $f(x) = 0$)	Symmetry with respect to y -axis or origin	Even or Odd Function $f(-x) = f(x)$ or $f(-x) = -f(x)$	Is the function periodic? If so, state the period.	Is $f(x)$ a one-to- one mapping? (For each $f(x)$ only one x exists)	State the x coordinates of any points of discontinuity
1. $f(x) = x$	$1\mathbb{R}$	$1\mathbb{R}$	$x = 0$	origin	odd	no	yes	none
2. $f(x) = x^2$	$1\mathbb{R}$	$y \geq 0$	$x = 0$	y -axis	even	no	no	none
3. $f(x) = x^3$	$1\mathbb{R}$	$1\mathbb{R}$	$x = 0$	origin	odd	no	yes	none
4. $f(x) = x $	$1\mathbb{R}$	$y \geq 0$	$x = 0$	y -axis	even	no	no	none
5. $f(x) = [x]$	$1\mathbb{R}$	$y = k,$ $k \in \mathbb{J}$	$0 \leq x < 1$	neither	neither	no	no	$x = k, k \in \mathbb{J}$
6. $f(x) = \sin x$	$1\mathbb{R}$	$ y \leq 1$	$x = k\pi,$ $k \in \mathbb{J}$	origin	odd	2π	no	none
7. $f(x) = \cos x$	$1\mathbb{R}$	$ y \leq 1$	$x = (2k + 1)\pi/2,$ $k \in \mathbb{J}$	y -axis	even	2π	no	none
8. $f(x) = \tan x$	$x \in 1\mathbb{R},$ $x \neq (2k + 1)\pi/2,$ $k \in \mathbb{J}$	$1\mathbb{R}$	$x = k\pi,$ $k \in \mathbb{J}$	origin	odd	π	no	$x = (2k + 1)\pi/2, k \in \mathbb{J}$
9. $f(x) = \sec x$	$x \in 1\mathbb{R},$ $x \neq (2k + 1)\pi/2,$ $k \in \mathbb{J}$	$ y \geq 1$	none	y -axis	even	2π	no	$x = (2k + 1)\pi/2, k \in \mathbb{J}$
10. $f(x) = 2^x$	$1\mathbb{R}$	$y > 0$	none	neither	neither	no	yes	none
11. $f(x) = \log_2 x$	$x > 0$	$1\mathbb{R}$	$x = 1$	neither	neither	no	yes	none
12. $f(x) = 1/x$	$x \in 1\mathbb{R},$ $x \neq 0$	$y \in 1\mathbb{R},$ $y \neq 0$	none	origin	odd	no	yes	$x = 0$
13. $f(x) = 1/x^2$	$x \in 1\mathbb{R},$ $x \neq 0$	$y > 0$	none	y -axis	even	no	no	$x = 0$
14. $f(x) = \sqrt{x}$	$x \geq 0$	$y \geq 0$	$x = 0$	neither	neither	no	yes	none
15. $f(x) = \sqrt{a^2 - x^2}$	$ x \leq a$	$0 \leq y \leq a$	$x = \pm a$	y -axis	even	no	no	none
16. $x = \begin{cases} 0, & x \text{ is rational} \\ 1, & x \text{ is irrational} \end{cases}$	$1\mathbb{R}$	$y = 1$ or $y = 0$	all rational numbers	y -axis	even	no	no	Every point is a point of discontinuity.

Section III

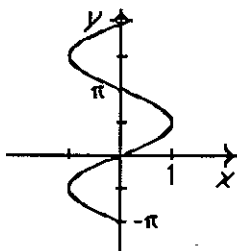
1. In an even function, $f(-x) = f(x)$, which means that if $f(x)$ contains point $(x, f(x))$, then it contains point $(-x, f(x))$, therefore, symmetry is across the y-axis. In an odd function, $f(-x) = -f(x)$, which means that if $f(x)$ contains point $(x, f(x))$, then it contains $(-x, -f(x))$, therefore symmetry is across the origin.

2. c, d

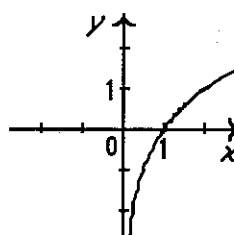
a.



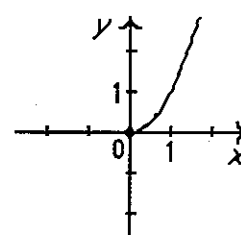
b.



c.



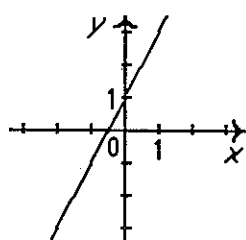
d.



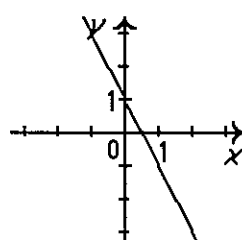
3. If the function is a one-to-one mapping (for each x there is only one $f(x)$ and vice versa) then the reflection across $y = x$ is a function.

Worksheet 2

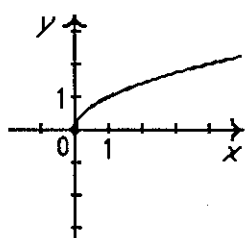
1. a.



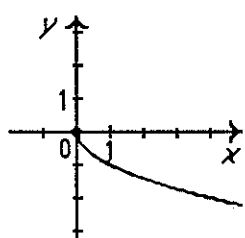
b.



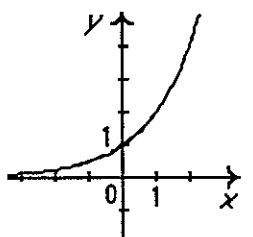
2. a.



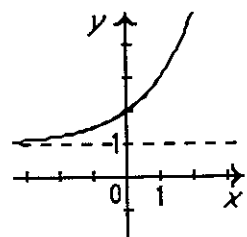
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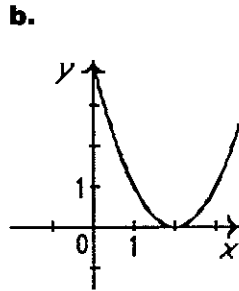
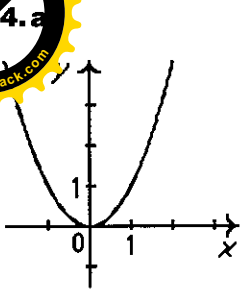


3. a.

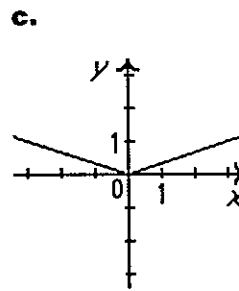
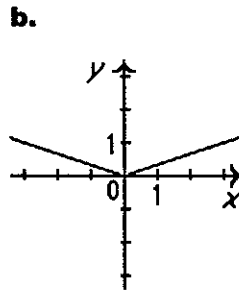
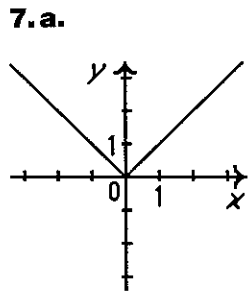
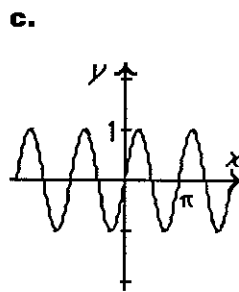
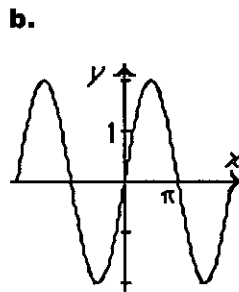
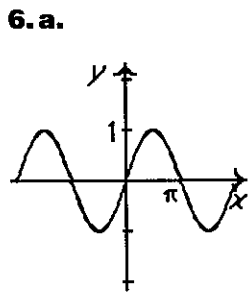


b.





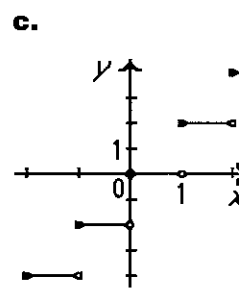
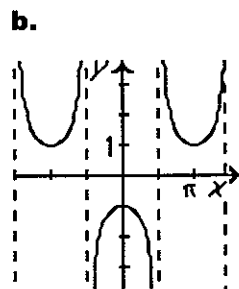
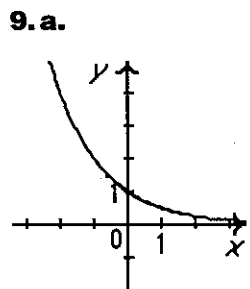
- 5.a.** reflection of the graph of $f(x)$ across the y -axis
b. reflection of the graph of $f(x)$ across the x -axis
c. vertical translation of the graph of $f(x)$ up c units for $c > 0$, vertical translation of the graph of $f(x)$ down c units for $c < 0$
d. horizontal translation of the graph of $f(x)$ $|c|$ units to the right for $c > 0$, horizontal translation of the graph of $f(x)$ $|c|$ units to the left for $c < 0$



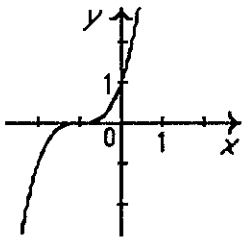
- 8.a.** "steeper" graph, each range value increased by a factor of c for $c > 1$; "less steep" graph, each range value decreased by a factor of c for $0 < c < 1$.

NOTE: Range behavior of f over an interval I now is repeated in $\frac{1}{c}$ times the width of interval I . If f is periodic, of period k , then $f(cx)$ will have period $\frac{k}{c}$.

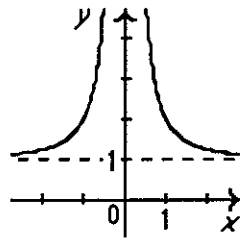
- b.** shortens domain for same function behavior for $c > 1$ lengthens domain for same function behavior for $c < 1$.



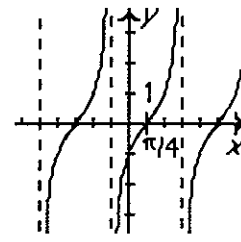
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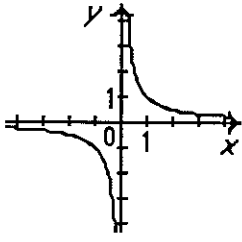
e.



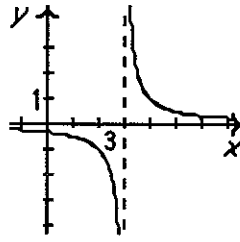
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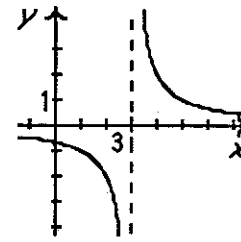
10.a. $f_1(x) = \frac{1}{x}$



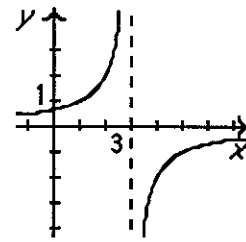
$f_2(x) = \frac{1}{x-3}$



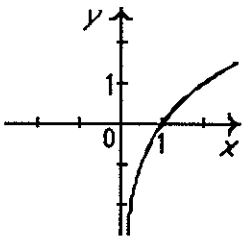
$f(x) = \frac{2}{x-3}$



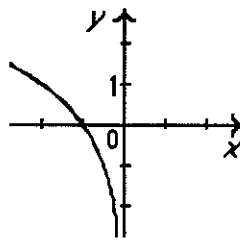
$f_4(x) = \frac{-2}{x-3}$



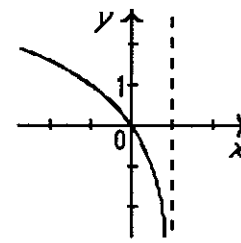
b. $f_1(x) = \log_2 x$



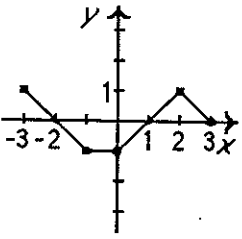
$f_2(x) = \log_2(-x)$



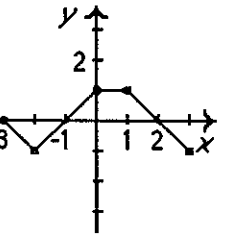
$f_3(x) = \log_2(1-x)$



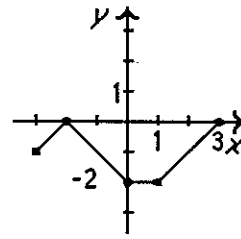
11.a.



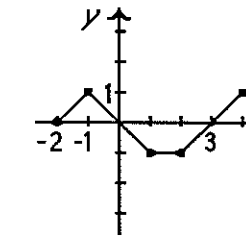
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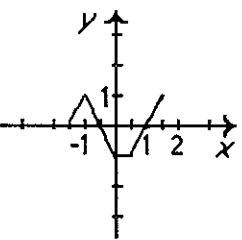
c.



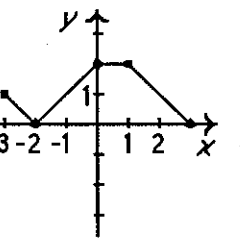
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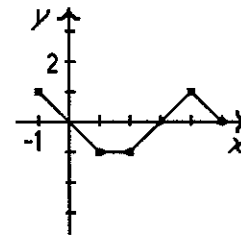
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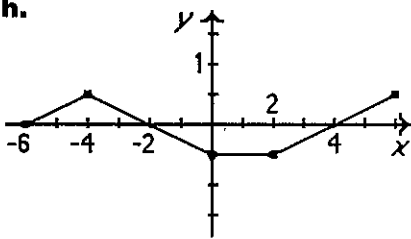
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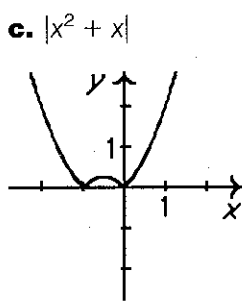
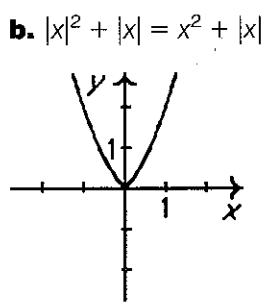
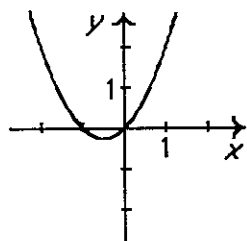
g.



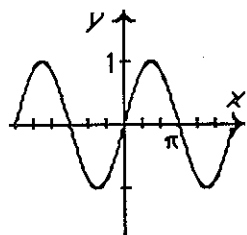
h.



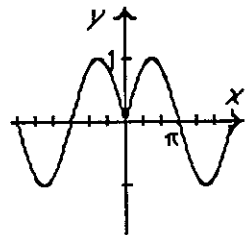
Worksheet 3



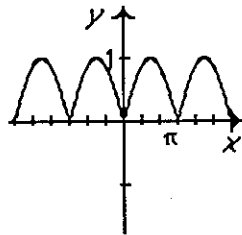
2. a.



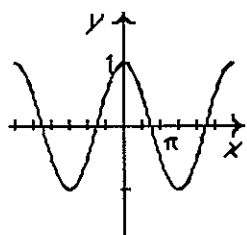
b. $\sin|x|$



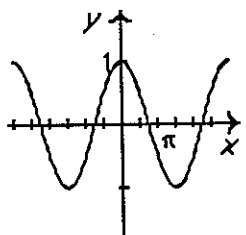
c. $|\sin x|$



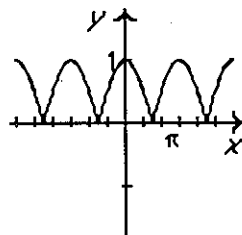
3. a.



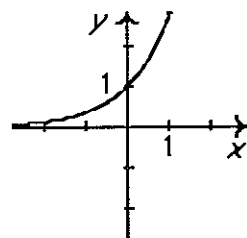
b. $\cos|x|$



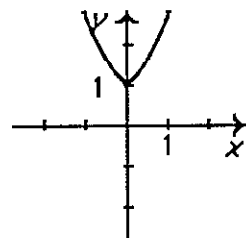
c. $|\cos x|$



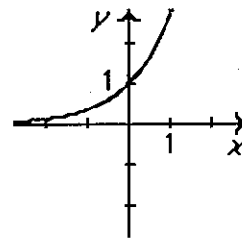
4. a.



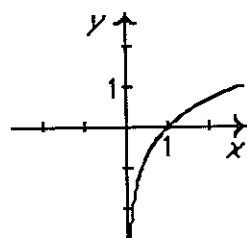
b. $e^{|x|}$



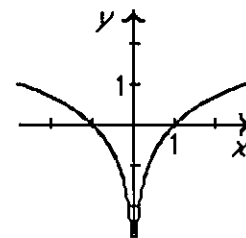
c. e^x



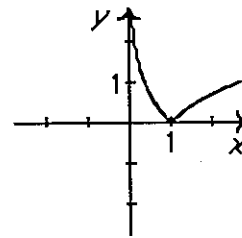
5. a.



b. $\ln|x|$



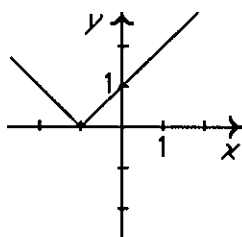
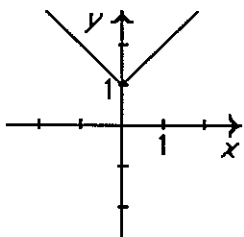
c. $|\ln x|$



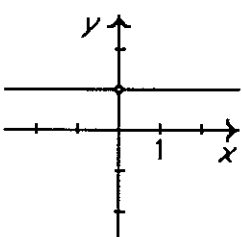
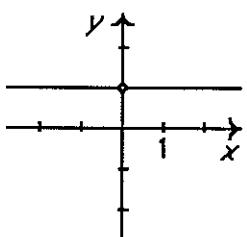
6. a. The graph of $f(x)$ for $x > 0$ remains the same and then reflects itself across the y -axis to complete the graph for $x < 0$.

- b. Any points of the graph of $f(x)$ where $f(x) < 0$ will be reflected across the x -axis. The remainder of the graph of $f(x)$ remains unchanged.

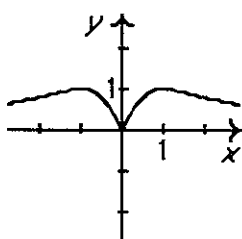
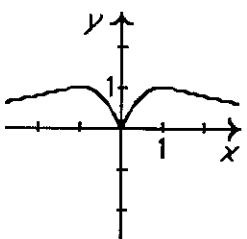
7.



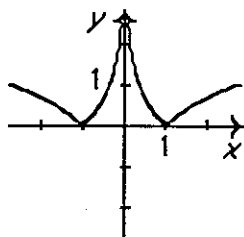
8.



9.



10.



11. all even functions or all functions $f(x)$ whose graphs are symmetric across the y -axis
12. all functions $f(x)$ with $f(x) \geq 0$
13. even functions with $f(x) \geq 0$
14. A, B, E
15. A, B, D