Wednesday, FEBRUARY 27, 2002

Contest B

The MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

Presented by the Akamai Foundation

AMC 12

53rd Annual American Mathematics Contest 12

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question, multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** working time to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 20th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 26, 2002 or Tuesday, April 9, 2002. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. The arithmetic mean of the nine numbers in the set $\{9,99,999,9999,\ldots,999999999\}$ is a 9-digit number M, all of whose digits are distinct. The number M does not contain the digit

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

2. What is the value of

$$(3x-2)(4x+1)-(3x-2)4x+1$$

   when $x=4$?

   (A) 0 **(B)** 1 **(C)** 10 **(D)** 11 **(E)** 12

3. For how many positive integers n is n^2-3n+2 a prime number?

   (A) none **(B)** one **(C)** two **(D)** more than two, but finitely many
   (E) infinitely many

4. Let n be a positive integer such that $\frac{1}{2}+\frac{1}{3}+\frac{1}{7}+\frac{1}{n}$ is an integer. Which of the following statements is **not** true:

   (A) 2 divides n **(B)** 3 divides n **(C)** 6 divides n **(D)** 7 divides n
   (E) $n>84$

5. Let v, w, x, y, and z be the degree measures of the five angles of a pentagon. Suppose $v<w<x<y<z$ and v, w, x, y, and z form an arithmetic sequence. Find the value of x.

   (A) 72 **(B)** 84 **(C)** 90 **(D)** 108 **(E)** 120

6. Suppose that a and b are nonzero real numbers, and that the equation $x^2+ax+b=0$ has solutions a and b. Then the pair (a,b) is

   (A) $(-2,1)$ **(B)** $(-1,2)$ **(C)** $(1,-2)$ **(D)** $(2,-1)$ **(E)** $(4,4)$

7. The product of three consecutive positive integers is 8 times their sum. What is the sum of their squares?

   (A) 50 **(B)** 77 **(C)** 110 **(D)** 149 **(E)** 194

8. Suppose July of year N has five Mondays. Which of the following must occur five times in August of year N? (Note: Both months have 31 days.)

   (A) Monday **(B)** Tuesday **(C)** Wednesday **(D)** Thursday **(E)** Friday

9. If a, b, c, d are positive real numbers such that a, b, c, d form an increasing arithmetic sequence and a, b, d form a geometric sequence, then $\frac{a}{d}$ is

(A) $\frac{1}{12}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

10. How many different integers can be expressed as the sum of three distinct members of the set $\{1, 4, 7, 10, 13, 16, 19\}$?

(A) 13 **(B)** 16 **(C)** 24 **(D)** 30 **(E)** 35

11. The positive integers A, B, $A - B$, and $A + B$ are all prime numbers. The sum of these four primes is

(A) even **(B)** divisible by 3 **(C)** divisible by 5 **(D)** divisible by 7 **(E)** prime

12. For how many integers n is $\frac{n}{20-n}$ the square of an integer?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 10

13. The sum of 18 consecutive positive integers is a perfect square. The smallest possible value of this sum is

(A) 169 **(B)** 225 **(C)** 289 **(D)** 361 **(E)** 441

14. Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 16

15. How many four-digit numbers N have the property that the three-digit number obtained by removing the leftmost digit is one ninth of N?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

16. Juan rolls a fair regular octahedral die marked with the numbers 1 through 8. Then Amal rolls a fair six-sided die. What is the probability that the product of the two rolls is a multiple of 3?

(A) $\frac{1}{12}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{7}{12}$ **(E)** $\frac{2}{3}$

17. Andy's lawn has twice as much area as Beth's lawn and three times as much area as Carlos' lawn. Carlos' lawn mower cuts half as fast as Beth's mower and one third as fast as Andy's mower. If they all start to mow their lawns at the same time, who will finish first?

(A) Andy **(B)** Beth **(C)** Carlos **(D)** Andy and Carlos tie for first.

(E) All three tie.

18. A point P is randomly selected from the rectangular region with vertices $(0,0), (2,0)$, $(2,1), (0,1)$. What is the probability that P is closer to the origin than it is to the point $(3,1)$?

(A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{4}{5}$ **(E)** 1

19. If a,b, and c are positive real numbers such that $a(b+c) = 152$, $b(c+a) = 162$, and $c(a+b) = 170$, then abc is

(A) 672 **(B)** 688 **(C)** 704 **(D)** 720 **(E)** 750

20. Let $\triangle XOY$ be a right-angled triangle with $m\angle XOY = 90^\circ$. Let M and N be the midpoints of legs OX and OY, respectively. Given that $XN = 19$ and $YM = 22$, find XY.

(A) 24 **(B)** 26 **(C)** 28 **(D)** 30 **(E)** 32

21. For all positive integers n less than 2002, let

$$a_n = \begin{cases} 11, & \text{if } n \text{ is divisible by 13 and 14;} \\ 13, & \text{if } n \text{ is divisible by 14 and 11;} \\ 14, & \text{if } n \text{ is divisible by 11 and 13;} \\ 0, & \text{otherwise.} \end{cases}$$

Calculate $\displaystyle\sum_{n=1}^{2001} a_n$.

(A) 448 **(B)** 486 **(C)** 1560 **(D)** 2001 **(E)** 2002

22. For all integers n greater than 1, define $a_n = \dfrac{1}{\log_n 2002}$. Let $b = a_2+a_3+a_4+a_5$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then $b - c$ equals

(A) -2 **(B)** -1 **(C)** $\dfrac{1}{2002}$ **(D)** $\dfrac{1}{1001}$ **(E)** $\dfrac{1}{2}$

23. In $\triangle ABC$, we have $AB = 1$ and $AC = 2$. Side $\overline{BC}$ and the median from A to $\overline{BC}$ have the same length. What is BC?

(A) $\dfrac{1+\sqrt{2}}{2}$ **(B)** $\dfrac{1+\sqrt{3}}{2}$ **(C)** $\sqrt{2}$ **(D)** $\dfrac{3}{2}$ **(E)** $\sqrt{3}$

24. A convex quadrilateral $ABCD$ with area 2002 contains a point P in its interior such that $PA = 24$, $PB = 32$, $PC = 28$, and $PD = 45$. Find the perimeter of $ABCD$.

(A) $4\sqrt{2002}$ **(B)** $2\sqrt{8465}$ **(C)** $2\left(48+\sqrt{2002}\right)$

(D) $2\sqrt{8633}$ **(E)** $4\left(36+\sqrt{113}\right)$

25. Let $f(x) = x^2+6x+1$, and let R denote the set of points (x, y) in the coordinate plane such that

$$f(x) + f(y) \le 0 \quad \text{and} \quad f(x) - f(y) \le 0.$$

The area of R is closest to

(A) 21 **(B)** 22 **(C)** 23 **(D)** 24 **(E)** 25

This page left intentionally blank.

2002

AMC 12 - Contest B

DO NOT OPEN UNTIL

Wednesday, FEBRUARY 27, 2002

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 27.** Nothing is needed from inside this package until February 27.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form B found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC Director, Titu Andreescu, no later than 24 hours following the examination.
4. <u>Please Note:</u> All Problems and Solutions are copyrighted; it is illegal to make copies or transmit them on the internet or world wide web without permission.
5. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Duplication at any time via copier, telephone, eMail, World Wide Web or media of any type is a violation of the copyright law.*

Sponsored by
The MATHEMATICAL ASSOCIATION OF AMERICA
The Akamai Foundation
University of Nebraska – Lincoln
Contributors
American Statistical Association
Casualty Actuarial Society
Society of Actuaries
National Council of Teachers of Mathematics
American Society of Pension Actuaries
American Mathematical Society
American Mathematical Association of Two Year Colleges
Pi Mu Epsilon
Consortium for Mathematics and its Applications
Mu Alpha Theta
National Association of Mathematicians
Kappa Mu Epsilon
School Science and Mathematics Association
Clay Mathematics Institute
Institute for Operations Research and the Management Sciences
Canada/USA Mathpath & Mathcamp

WRITE TO US!

Correspondence about the problems and solutions for this ***AMC 12*** *should be addressed to:*

Prof. David Wells, Department of Mathematics
Penn State University, New Kensington, PA 15068
Phone: 724/334-6749; Fax: 724/334-6110; email: dmw8@psu.edu

Orders for any of the publications listed below should be addressed to:

Titu Andreescu, Director
American Mathematics Competitions
University of Nebraska, P.O. Box 81606
Lincoln, NE 68501-1606
Phone: 402-472-2257; Fax: 402-472-6087; email: titu@amc.unl.edu;

2002 AIME

The AIME will be held on Tuesday, March 26, 2002 with the alternate on April 9,2002. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score in the top 1% of the AMC 10 or receive a score of 100 or above on the AMC 12. Alternately, you must be in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) on Thursday through Sunday, May 9-12, 2002 in Cambridge, MA. The best way to prepare for the AIME and USAMO is to study previous years of these exams. Copies may be ordered as indicated below.

PUBLICATIONS

MINIMUM ORDER: $10 (before shipping/handling fee), *PAYMENT IN US FUNDS ONLY made payable to the* American Mathematics Competitions or VISA/MASTERCARD/AMERICAN EXPRESS accepted. Include card number, expiration date, cardholder name and address. U.S.A. and Canadian orders must be prepaid and will be shipped Priority Mail, UPS or Air Mail.

INTERNATIONAL ORDERS: Do **NOT** prepay. An invoice will be sent to you.

Examinations: Each price is for one copy of an exam and its solutions for one year. Specify the years you want and how many copies of each. All prices effective to September 1, 2003.

- AMC 10 2000-2003/AHSME (AMC 12) 1989-2003, $1 per copy per year.
- AIME 1989-2003, $2 per copy per year (2003 available after April)
- USA and International Math Olympiads, 1989-1999, $5 per copy per year; 2000, 2001 - $14.00 each
- National Summary of Results and Awards, 1989-2003, $10 per copy per year.
- Problem Book I, AHSMEs 1950-60, Problem Book II, AHSMEs 1961-65, $10/ea
- Problem Book III, AHSMEs 1966-72, Problem Book IV, AHSMEs 1973-82, $13/ea
- Problem Book V, AHSMEs and AIMEs 1983-88, $30/ea
- Problem Book VI, AHSMEs 1989-1994, $24/ea
- USA Mathematical Olympiad Book 1972-86, $18/ea
- International Mathematical Olympiad Book II, 1978-85, $20/ea
- World Olympiad Problems/Solutions 1995-96, 1996-97, 1997-98, $15/ea
- Mathematical Olympiads Problems & Solutions from around the World 1998-1999, 1999-2000, $25/ea
- The Arbelos, Volumes I-V, and a Special Geometry Issue, $8/ea

Shipping & Handling charges for Publication Orders:

Order Total	Add:	Order Total	Add:
$ 10.00 -- $ 30.00	$ 5	$ 40.01 -- $ 50.00	$ 9
$ 30.01 -- $ 40.00	$ 7	$ 50.01 -- $ 75.00	$12
	$ 75.01 -- up	$15	